UNCAPACITATED WAREHOUSE LOCATION WITH PROBABILISTIC DEMAND

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By
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TO MY

RESPECTED PARENTS

CERTIFICATE

Certified that this work on 'Uncapacitated Warehouse Location with Probabilistic Demand' by Vasdev Chanchlani, has been carried out under my supervision and that this has not been submitted elsewhere for award of a degree.

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Vasdev Chanchlani

CONTENTS

ABSTRACT

The present work describes a methodology for solving the uncapacitated warehouse location problem considering demands as independent random variables belonging to a known 2-parameter distribution. The objective is to minimize total fixed and variable transportation costs. The problem is initially structured as a chance constrained problem assuming that the demands are normally distributed. The chance constrained programming problem is converted into a deterministic mixed integer programming model. The mixed integer programming model is solved to determine if all the decision variable are integers. Integer values for all the decision variable indicate that an optimal solution to the problem has been obtained. If all the decision variables are not integer the problem is solved using branch and bound concepts. An efficient branch and bound algorithm involving three node simplification procedures and eight branching rules has been presented to arrive at the optimal solution. The branch and bound algorithm used is tested using a problem from the literature. The computational efficiency of the algorithm has been evaluated based on the computational experience of solving 20 problems. In all,

problems of four different sizes are considered and for each size five problems are solved. It is found that the computational time does not increase drastically with the increase in the size of the problem.

A computer package for the proposed branch and bound procedure has been developed for the IBM 7044/1401 system. The package in its present form can handle problems involving 50 warehouses and 100 customers. A procedure for the selection of optimum confidence level with which the consumer demand will be satisfied has been developed and illustrated with the help of a numerical example.

CHAPTER I

INTRODUCTION

Management of industrial logistics seeks to maximize the economic value of products or materials by getting and having the products where they are wanted, at the time they are wanted and at a reasonable cost. The management of physical supply and distribution forms an important subsystem of the overall domain of the management of industrial logistics and considerably influences the cost of the product/ commodity to the consumers. The design of a physical distribution system involves the selection of the number, size and location of warehouses, the mode of transportation from plant to the warehouses and from warehouses to the customers. It is interesting to point out that operation of warehouses and carrying cost of material stored in them account for an amount equivalent to 6 percent of gross national product. On the individual firm level the cost of physical distribution of the product may be as high as 15 percent of the total expenditure. These figures reflect the importance of designing an efficient-distribution system.

During the past few years a large number of organizations, attracted by the possibility of making substantial savings, through the use of efficient distribution systems, have been critically examining in warehousing policy. Since the selection of location and size of the warehouses considerably influences the cost of distribution, a lot of attention is currently being devoted to this specific area.

There are a host of tangible and intangible factors which influence the location of a warehouse. Till the emergence of operation research techniques, the problem lay mainly in the hands of economists who handled it primarily using qualitative approaches. However, there are certain quantitative aspects of the problem which cannot be reckoned accurately through intuition alone. Since 1958, considerable research effort seems to have been deployed for the development of quantitative approaches for warehouse location problems. Though, today quantitative techniques are heavily emphasised, one should not overlook the importance of qualitative aspects of this problem. It should be realized that analytical approaches yield a solution to the model but not necessarily to the problem. The solutions obtained through analysis serve only as an aid in decision making. There remain a number of non-quantifiable factors which must be considered alongwith analytical results in making

the transition from the model to the problem. In some cases the solution obtained from analysis will appear impractical from an operational point of view. Such solutions should be interpreted as bench marks against which operationally accepted solutions can be compared.

The selection of the set of potential warehouse locations depends on many tangible and intangible factors. Important factors which influence the location of a warehouse are: market orientation, communication and transportation facilities, nature of product, local considerations like site value, attitude of local government including the planning restriction, labour availability, ancillary services, etc.

1.1 Classification of Warehousing Problems:

Warehousing problems can be classified based on warehouse capacity, cost of opening the warehouse and customer's demand characteristics. The variations of the problem in these three basic classes are given below:

1. Warehouse Capacity:

Based on the capacity of warehouse, the problem is referred to as capacitated or uncapacited warchousing problem. In the capacitated problem, the warehouse capacity is assumed to be fixed while in uncapacited problem it is assumed to be infinite.

2. Cost of Opening the Warehouse:

If there is no fixed cost associated with the opening of the warehouse, then the problem is known as 'Depot Location Problem'. On the other hand, there may be fixed costs of opening each of the warehouses. In such a case we have to determine the optimum number of warehouses and their locations such that the overall cost of transportation and fixed charges of opening the warehouses is minimized. This is known as fixed charge warehouse problem.

3. Consumer's Demand:

Depending upon the type of customer's demand the problem can be classified as Deterministic Demand Problem or Probabilistic Demand Problem.

The above stated classification scheme is depicted in Figure 1.

1.2 Motivation:

A considerable amount of work has been done by previous researchers on the fixed charge warehouse location problem. Various analytical and heuristic approaches have been developed for this problem when the demand is known. However, no attention seems to have been given to the case when demands are probabilistic. It was thus felt that structuring

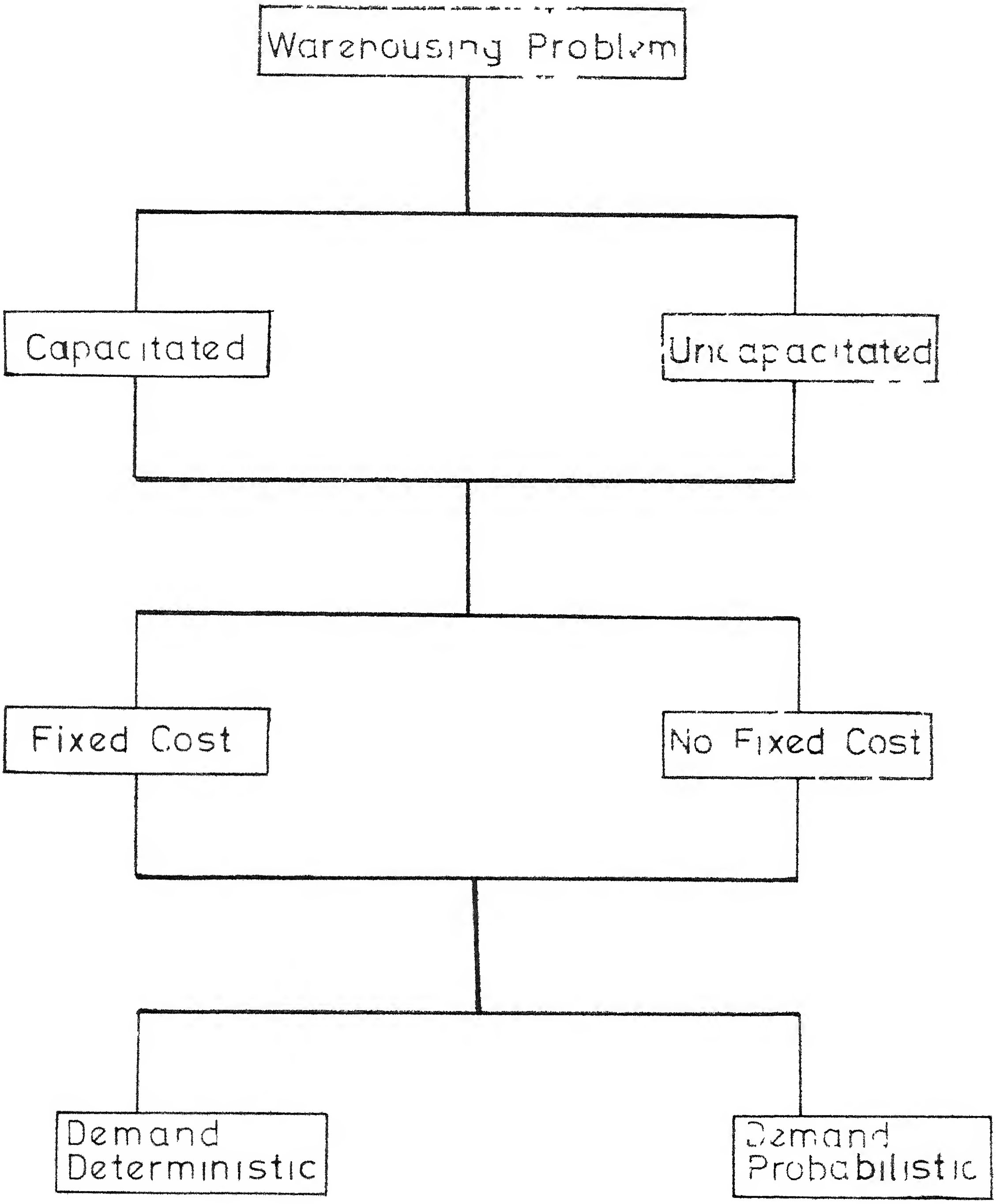


FIG 1. CLASSIFICATION OF WAREHOUSING PROBLEM

the model to accommodate a probabilistic demand constraint would be worthwhile effort. Therefore, the present work is directed towards the optimal location of warehouses in a given system when demand is considered as probabilistic. The problem in its simplest form can be stated as: given a number of demand points where the demand from the customers are concentrated. The demands are assumed to be probabilistic and are supplied from warehouses which are to be located at a given sets of sites. The objective is to determine the following such that the total cost which comprises of transportation and fixed costs of operating the system is minimized:

(a) The optimum number of warehouses to be opened.

(b) The location for the warehouses to be opened (chosen from a given set of alternative sites)

(c) The set of demand points to be served by a given warehouse.

To seek logical answers to the above problem, an analytical model has been formulated. The stochastic nature of demand is accounted for by structuring the model as a chance constrained programming problem. The probabilistic constraints are reduced to deterministic constraints by using the normal distribution. The basic philosophy of the branch and bound procedure has been utilized to solve the

problem. The various branching rules have been used to arrive at the optimal solution.

A computer package for the solution methodology has been developed in FORTRAN IV for the IBM 7044 system. The computer package is quite general in nature and in its present form can handle problems involving 50 warehouses and 100 customers.

1.3 Organisation of the Thesis:

In addition to this introductory chapter, there are three more chapters. In Chapter II, a brief survey of the existing literature relevant to the kind of the problems with which this thesis is concerned; is given.

Chapter III is divided into three parts. In part 1 the problem is formulated. In part 2, solution methodology is presented to solve the problem. In part 3, an example is considered to demonstrate the procedure.

In the last chapter, computational experience with problems of different sizes is presented. Further extension of the problem and suggestions for future work are also described in this chapter.

CHAPTER II

LITERATURE REVIEW

A review of the literature on location theory indicates that considerable research effort has been devoted to the fixed charge location problem. The problem in its simplest form involves the shipment of a single commodity to n customers with known demand from m potential warehouses. The warehouses which are opened for making the shipments incur certain fixed costs. The objective is to select from the potential warehouses, which should be opened to minimize the total cost, i.e., transportation costs and fixed costs of operating system.

The current state of art regarding the approaches available for solving this problem can broadly be categorized as follows:

(1) Heuristic Approaches,

(2) Simulation and Graph Theoretic Techniques,

(3) Branch and Bound Approaches,

(4) Network and Mathematical Methods.

Discussion of these approaches will be limited to some of the important contributions

2.1 Heuristic Approaches:

The difficulty encountered in working with heuristics is that there is no way to know how far the answer obtained is from the optimal. This difficulty tends to decrease the validity of a sensitivity analysis by obscuring the cause of a change in the solution which could occur if a particular parameter is varied. For example, results from a particular procedure may be same as optimal for cases in which the fixed costs for operating a warehouse are large relative to the variable costs, but when that ratio is reduced the solutions produced move away from the optimal solutions. Since we may be interested in the sensitivity of the solutions to changes in this ratio as a part of the general sensitivity analysis, the conclusions from using the heuristic procedure will be confounded by the fact that the quality of solution is changing as well as the solutions themselves. Inspite of the above difficulties, the heuristic procedures still offer several advantages. The advantage of speed is a major consideration and the comparisons with exact solutions have indicated that this is not gained at the expense of a large penalty in terms of quality of solutions. In addition, heuristic procedures are versatile, sometimes more so than problem specific exact procedures such as we have in case of the warehouse location problem. This may mean that

additional factors or restrictions can be incorporated more easily into the heuristic procedures than the exact procedures.

Kuehn and Hamburger [12] have developed a heuristic or non-optimal algorithm to find good solutions to the warehousing problem. They assume that transport costs are linear with the amount shipped and facility costs are of the form,

$$F_i(y_i) = a_i + b_i y_i \qquad \text{if the facility exists,}$$

$$= 0 \qquad \text{if it does not exist.}$$

Thus $F_i(y_i)$ consists of fixed charge that is independent of storage and a linear cost which does depend on the amount of storage if the facility exists. Since the expansion cost b_i is assumed for each facility, the demand points which a given facility will serve is determined simply by assigning a demand point to that facility for which the sum of shipment cost and expansion cost is minimum.

The solution method begins with a single facility, and another facility is added to see if the total cost can be decreased. The assumption is that the best N facilities are contained in the set N + 1 facilities. Termination of solution occurs when it appears that another facility cannot

be added without increasing the total cost, and those warehouses which have become uneconomical are eliminated. A warehouse becomes uneconomical if a subsequently added facility can serve enough of its customers at lower cost.

Feldman, Lehrer, and Ray [5] have assumed a more general form for the facility cost (continuous concave function). They assume, too, that the transport cost is linear in the amount shipped between points. Since $F_i(Y_i)$ does not have an expansion cost which is linear with the size of the facility, the problem of assigning demand points to existing facilities is much more difficult to accomplish than before. Whereas Kuehn and Hamburger begin with one facility, Feldman et al start with all facilities existing and drop out uneconomical warehouses. Termination occurs when no further savings can be achieved by elimination of any warehouse.

Drysdale and Sandiford [3] have suggested a heuristic which occupies a middle ground between precise analytical modes and simulation. It uses a 'dropping heuristic' combined with steadily incrementing fixed warehouse cost as well as a local enumeration after each 'dropping iteration'. This approach offers certain advantages over rigorous methods, in terms of greater computational speed and flexibility in the form of the cost function. However, the procedure lacks the

ability to reach with certainty the precise optimum.

2.2 Simulation and Group Theoretic Techniques:

Simulation has been used by Markland [15] and Shycon and Maffei [19] for warehouse location problems. The procedure essentially procedure a total cost curve as a function of the number of warehouses that are to be included in a physical distribution system. Samples of specific warehouses are chosen and to each warehouse those customers are assigned which can be supplied at the lowest variable cost and then total costs are evaluated. Khumawala remarks [9] that in using the simulation approach, the need to solve the transportation programs increase the computation time for getting results.

A group theoretic approach has been suggested by Kennington and Unger [10]. They have found that associated group problem can be especially structured with predictable inter-relationship among the group variables. The dependency of sub-group order on the route capacity is also described. Authors claim that by taking advantage of group theoretic properties of the problem the exact optimal solution can always be found.

2.3 Branch and Bound Approaches:

The branch and bound algorithm developed by Little, et al [13] for travelling salesman problem was the starting point in the development of many branch and bound algorithms, for warehouse locational problem. Notable contribution have been made by Efroymson and Ray [7], Spielberg [18] and Khumawala [7,8]. All these investigators have used the same philosophy for search in the use of branch and bound for solving warehouse locational problem. The efficiency of a branch and bound procedure depends mainly on the determination of the lower bounds. The bounding procedures proposed are based on minimum total cost with integer restriction on the number of warehouses to be opened.

The branch and bound procedure requires generating of a sequence of partial assignments and analyzing the completion of each in search of successively better feasible solutions. Efroymson and Ray's method involves selective enumeration which is guided at each stage by a bound on the value of the objective function obtained at that stage. The efficiency of branch and bound in his formulation is due to the fact that non-integer result at each stage is an obvious solution to a simple linear programming problem. The algorithm terminates when there are no nodes (stages) left which can possibly produce an improved solution. A hidden

advantage in the formulation, in addition to the fact that it terminates optimally, is that the solution is independent of all linearities in the transportation cost function.

Spielberg [18] has extended Efroymson and Ray's branch and bound algorithm by incorporating features to attain better computational efficiency. In this paper, the author has reported his computational experience for varied sizes of problems. The largest problem considered involves 100 warehouses and 150 customers. Spielberg has also suggested models to incorporate linear facility expansion cost and budget constraint on total expenditure. However, he has not reported any computational experience.

Recently Khumawala [7] has developed various branching rules. These rules are largest Omega rule, Smallest Omega rule, Largest Delta rule, Smallest Delta rule, Largest 'Y' rule, Smallest 'Y' rule , Largest Demand rule and Smallest Demand rule. These rules reduce the size of the branch and bound decision tree and thus lead to considerable saving from computational standpoint. By using the information already available at each node through node simplification criteria the author has suggested an improvement in solving the linear program without any computational work. The computational experience and efficiencies of various branching rules have also been reported with large number of problems.

2.4 Graphical and Mathematical Methods:

Marks [16] has formulated a fixed charge problem which is more general in nature. His model allows the facilities to be constrained in their capacity, and the warehouse is considered as an intermediate point between the sources of product and the demand points for the product. The facility cost function has a fixed charge and a linear expansion cost. If the source or demand points do not exist, the problem reduces to the general warehousing model.

According to Mark's model, product is to be provided in specific quantities to each source area of finite supply. In mathematical form, the problem can be expressed as,

$$\text{Minimize:} \quad \sum_{i=1}^{m} F_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} \sum_{k=1}^{p} C_{ki} X_{ki}$$

Subject to the constraints:

$$\sum_{i=1}^{m} X_{ki} \leq S_k \qquad k = 1, 2, \ldots, p$$

$$\sum_{j=1}^{n} X_{ij} = \sum_{k=1}^{p} X_{ki} \qquad i = 1, 2, \ldots, m$$

$$\sum_{k=1}^{p} X_{ki} \leq Q_i y_i \qquad i = 1, 2, \ldots, m$$

$$D_j^u \geq \sum_{i=1}^{m} X_{ij} \geq D_j^l \qquad j = 1, 2, \ldots, n$$

$X^{*}_{i\ j}$, $X^{*}_{k\ i}{}^{*}$ are non-negative integer, $y_i = (0, 1)$

where,

y_i = 1 if the i-th facility is built

= 0 otherwise

$X^{*}_{i\ j}$ = flow of material from facility i to sink j

$X^{*}_{k\ i}{}^{*}$ = flow of material from source k to intermediate point i

$C^{*}_{i\ j}$ = $C_{ij} + R$ = unit cost associated with a transfer of material from facility i to sink j

R_j = unit variable and ~~x~~associated with using sink j

$C^{*}_{k\ j}{}^{*}$ = $C'_{ki} + T_k + V_i$ = unit cost associated with transfer of material from source k to facility i

C'_{ki} = Unit shipping cost from source K to facility i

T_k = unit variable cost associated with using source k

V_i = Unit variable cost associated with using facility i

F_i = Fixed charge for establishing facility i

S_k = amount supplied at source k

D^u_j = upper bound on amount demanded at sink j

D_j^l = lower bound on amount demand at sink j

Q_i = capacity of the i-th facility

m = number of proposed facility sites

n = number of demand points

p = number of supply points

The solution technique is based on the recognition that a network may be applied to the problem. The graph representing the network is shown in Fig. 2. A capacitated node for each facility has been added so that a capacity constraint and a linear cost function may be ascribed to each facility.

The method begins by approximating the fixed charge cost function with the linear unit cost F_i/Q_i. The true cost function and the linear approximation cost function are shown in Fig. 3. It should be noted that the approximate cost function under-estimates the true cost function except when the flow through the facility is zero or Q_i, the capacity of the facility. The approximation forms the basis for a branch and bound scheme.

Assume all facilities are open and have the approximate cost function. Solve this initial problem using an out-of-kilter algorithm. If the resulting solution is such that flow through each of the facilities is either zero or

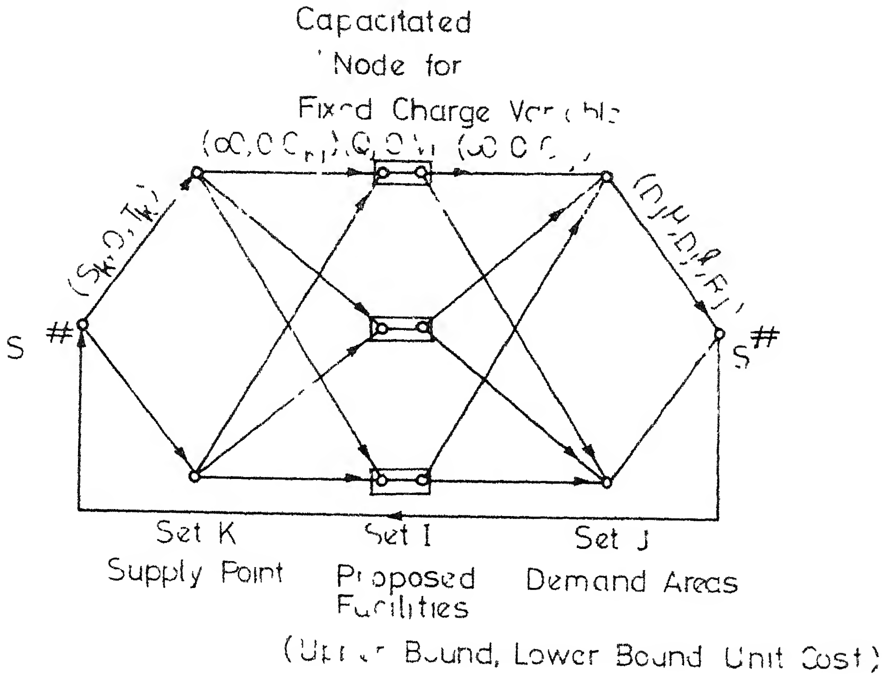


FIG 2 A GRAPH REPRESENTATION FOR A PROBLEM WITH $m=2$, $n=3$, $k=2$.

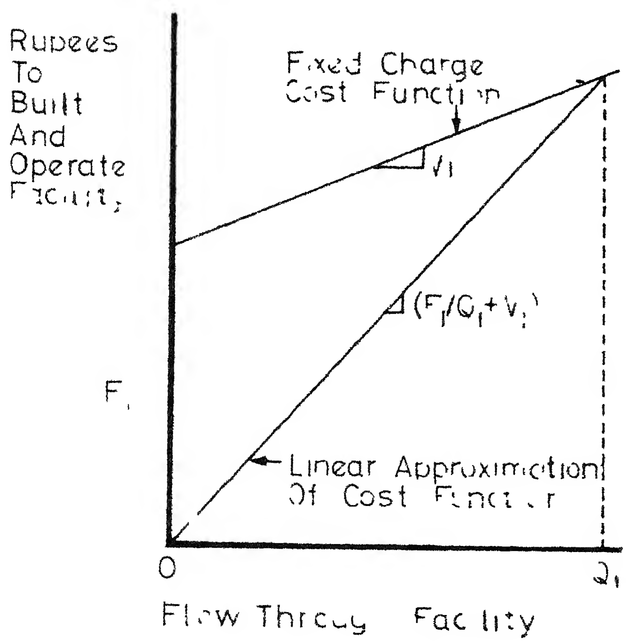


FIG 3 THE FIXED CHARGE COST FUNCTION AND ITS APPROXIMATION.

the capacity of the facility, the optimal solution to the fixed charge problem has been found. If not, branching takes place, and a facility is either included in the solution set with its fixed charge added to the cost or excluded from the solution by setting its capacity to zero. This procedure continues until an optimal solution is found and verified.

A recent and promising approach to the fixed charge problem has been presented by Walker and Lynn [20]. The thrust of their algorithm is to complement the evaluator of standard simplex procedure with the fixed charge associated with the entering vector and decrease the evaluator by the fixed charge associated with the existing vector. In addition, the fixed charges associated with the basic variables driven positive in the exchange must be added to the evaluator, and the fixed charges associated with basic vectors driven to zero substracted. Let

P_j = vector currently not in the basis

P_k = vector currently in the basis which is to be removed if P_j enters,

f_i = fixed cost associated with any vector P_i

θ_j = level at which vector P_j is to enter chosen by

$\theta_j = \operatorname{Min}_{i \in I} \left(\frac{\lambda_i}{X_{ij}}\right) = \frac{\lambda_k}{X_{kj}}$, $X_{ij} \geq 0$, I the set of basis vectors

$Z_j - C_j$ = the conventional simplex evaluator

Δ_j = the modified evaluator

S = the set of basic variables which are zero but become positive due to the exchanging and

T = the set of basic variables which were positive but become zero due to the exchange

The modified evaluator is then

$$\Delta_j = f_j - f_k - O_j(Z_j - C_j) + \sum_{i \in S} f_i - \sum_{i \in T} f_i$$

In a minimizing problem, a negative Δ_j indicates an improvement in the objective function on the appropriate exchange. If iteration are repeated until all Δ_j are positive, a local optimum will have been reached. Since better solutions may exist but not at adjacent extreme points the algorithm forces non-basis vectors into the basis even though their inclusion causes an increase in the objective. At the new extreme points, the algorithm again seeks favourable evaluators on the possibility that a new pattern of exchanges will lead to a better solution than the previous local optimum.

To summarize, the literature survey reveals that various analytical and heuristic approaches are available

for solving this problem when demand is known. However, no attention seems to have been given to the case when demands are probabilistic. Therefore present work is directed towards the optimal location of warehouses in a given system when demand is considered as probabilistic. In Chapter III the formulation and solution procedure of this problem are given.

CHAPTER III

PROBLEM FORMULATION AND SOLUTION PROCEDURE

3.1 Statement of the Problem:

We are given a number of demand points where the demands from the customers are concentrated. The demands are assumed to be probabilistic and are supplied from warehouses which are to be located at a given set of sites. The objective is to determine the following such that the total cost, which comprises of transportation and fixed costs of operating the system is minimized.

(a) The optimum number of warehouses to be opened.

(b) The location for the warehouse to be opened (chosen from a given set of alternative sites)

(c) The set of demand points to be served by a given warehouse.

3.2 Assumptions:

A mathematical model based on the following assumptions has been developed.

(i) Unit transportation costs are constant, i.e., they don't change with the volume transported and with time.

(ii) Warehousing fixed costs are not affected by demand

(iii) Each warehouse has unlimited capacity

(iv) The demands are concentrated at a known set of points or locations. Demand points and warehouse location can coincide.

(v) The demands are normally distributed random variables.

The following notation is used for development of the mathematical model.

3.3 Notation:

m = number of possible warehouse sites

n = total number of customers (demand points)

X_{ij} = fraction of the demand of customer i which is satisfied by the warehouse located at site j, $i = 1, \ldots, m$, $j = 1, \ldots, n$

F_i = fixed charge resulting from establishing a warehouse at i

t_{ij} = unit transportation and handling cost from warehouse i to customer j

C_{ij} = cost of supply the entire demand of customer j from a warehouse located at site i,

$$Y_i = \begin{cases} 1, & \text{if a warehouse is located at site } i \\ 0, & \text{otherwise} \end{cases}$$

P_i = the set of customers that can be supplied by a warehouse at i,

N_j = the set of warehouses that can supply customer j,

n_i = the total number of demand points in the set P_i,

r_j = demand of customer j (a normally distributed random variable).

D_j = demand of customer j which is actually satisfied,

α_j = minimum level of confidence with which customer's actual demand, i.e. r_j must be satisfied.

$X_{\alpha j}$ = abscissa associated with the left tail of the normal probability density function for the confidence level α_j,

$\mu(r_j)$ = mean of the random variables r_j,

$\sigma(r_j)$ = standard deviation of the random variable r_j

3.4 Problem Formulation:

Using the above notation the warehouse location problem stated can be formulated as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} F_i Y_i \qquad (1)$$

$$\text{S.t.} \quad \sum_{i \varepsilon N_j} X_{ij} = 1 \qquad d = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{j \varepsilon P_i} X_{ij} \leq nY_i \qquad i = 1, 2, \ldots, m \qquad (3)$$

$$X_{ij} \geq 0 \qquad \forall\, i \text{ and } j \qquad (4)$$

$$Y_i = (0, 1) \qquad \forall\, i \qquad (5)$$

$$P[D_j \rangle r_j] \geq \alpha_j \qquad (6)$$

Explanations for the logic used for the development of the above stated mathematical model are given below:

(i) The objective function gives the total cost of operating the system considering the cost of transportation and the fixed costs for the warehouses which are opened.

(ii) Constraint (2) indicates that the demand for customer j is satisfied by some combination of supplies from the warehouses,

(iii) Constraint (3) indicates that the sum of fractions of satisfied demands of the various customers supplied by the open warehouse located at site i cannot exceed the total number of customers. However, this sum will be zero when the warehouse is closed.

(iv) Constraint (4) represents a non-negativity restriction on the decision variables X_{ij}. X_{ij} will be zero when warehouse is closed.

(v) Equation (5) represents a zero-one type of restriction. Y_i is 1, if the i-th warehouse is open. otherwise Y_i is zero,

(vi) Constraint (6) gives the level of confidence with which the j-th customer's demand will be satisfied.

It is interesting to point out that if there were no fixed costs associated with the opening of warehouses the optimum solution to the problem would be locate a warehouse at every site. In such a case the problem reduces to a simple assignment problem and one is required to determine the warehouses which should supply a particular customer. On the other hand, if the transportation costs are ignored the best solution to the problem would be to locate one warehouse at the site with the smallest fixed cost. Thus we see that in the general problem where both the transportation costs and the fixed costs are considered a balance between these two types of costs need to sought so as to minimize the total cost of operating the system.

The mathematical formulation as given by (1) to (6) carries the structure of a chance-constrained programming

problem because the demand constraints given by (6) are probabilistic. The chance-constrained programming problem can be converted into deterministic problem using the following approach.

3.4.1 Reduction of Chance-Constrained Programming Problem to A Deterministic Problem:

Let r_j be a normally distributed random variable with $\mu(r_j)$ and σ_{r_j} the mean and standard deviation of r_j, respectively. The parameters $\mu(r_j)$, σ_{r_j} are assumed to be estimated from past experience. Then,

$$P\,(D_j \geq r_j) \;\geq\; \alpha_j$$

$$P\left[\frac{D_j - \mu(r_j)}{\sigma r_j} \;\geq\; \frac{r_j - \mu(r_j)}{\sigma r_j}\right] \;\geq\; \alpha_j$$

The terms on the left hand side of each of the bracketed inequalities are standardized random variables with zero mean and unitary standard deviations.

Let $X_{\alpha j}$ be the abscissa associated with the left tail of the standardized normal distribution function for probability level α_j, as illustrated in Fig. 4.

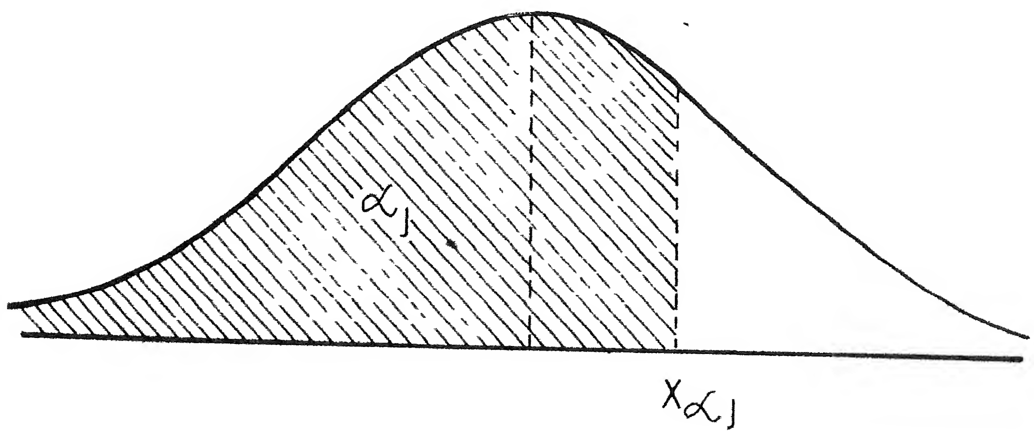


FIG. 4

Then,

$$P[X_{\alpha j} \geq \frac{r_j - \mu(r_j)}{\sigma r_j}] = \alpha_j$$

$$\Longrightarrow \frac{D_j - \mu(r_j)}{\sigma r_j} \geq X_{\alpha j} \quad \text{or}$$

$$D_j \geq X_{\alpha j} \, \alpha_{rj} + \mu \, (r_j)$$

Therefore for a given choice of the decision variable, D_j, we can write

$$P[D_j \geq r_j] \geq \alpha_j \quad \text{iff } D_j \geq X_{\alpha j} \, \sigma_{rj} + \mu(r_j) \tag{7}$$

Since there is no advantage in shipping more than the required amount, constraint (7) can be written as,

$$D_j = X_{\alpha j} \, \sigma r_j + \mu(r_j) \tag{8}$$

Substituting relation (7) for relation (6), the warehouse location and shipping problem can be written in its deterministic equivalent form without recourse to a probability for the constraint. The mathematical formulation of the stated problem reduces to,

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} (X\alpha_j \, \sigma r_j + \mu(r_j)) \, t_{ij} \, X_{ij} + \sum_{i=1}^{m} F_i \, Y_i \tag{9}$$

$$\text{S.t.} \quad \sum_{i \varepsilon N_j} X_{ij} = 1 \quad j = 1,2,\ldots,n \tag{10}$$

$$\sum_{j \varepsilon P_i} X_{ij} \leq nY_i \quad i = 1,2,\ldots,m \tag{11}$$

$$X_{ij} \geq 0 \quad \forall \; i \text{ and } j \tag{12}$$

$$Y_i = (0, 1) \quad \forall \; i \tag{13}$$

3.4.2 Necessary Conditions of Optimality:

From Appendix I (equations (18) to (22)) and equation (7), we have following relations,

$$\text{Minimize} \quad \hat{Z} = \sum_{i \varepsilon K_1 U K_2} \sum_{j=1}^{n} \left(t_{ij} \times D_j + \frac{g_i}{n_i} \right) X_{ij} \tag{14}$$

$$X_{ij} = 1 \quad j = 1,2,\ldots, n \tag{15}$$

$$X_{ij} \geq 0 \quad i \; \varepsilon \; K_1 U K_2, \quad j = 1,2,\ldots, n \tag{16}$$

$$D_j \geq X\alpha_j \, \sigma r_j + \mu(r_j) \tag{17}$$

The lagrangian expression for the problem can be given as,

$$L(D_j, \lambda_j) = \sum_{i \varepsilon K_1 U K_2} \sum_{j=1}^{n} (D_j \times t_{ij} + \frac{g_i}{n}) X_{ij}$$

$$- \sum_{j=1}^{n} \lambda_j (D_j - X\alpha_j \, \sigma r_j - \mu(r_j)$$

$$\frac{\partial L}{\partial D_j} = \sum_{i \varepsilon K_1 UK_2} t_{ij} X_{ij} - \lambda_j \geq 0$$

At the optimum value when we have decided to open the i^{th} warehouse we have,

$$t^*_{ij} - \lambda_j \geq 0$$

$$t^*_{ij} \geq \lambda_j \qquad (18)$$

If λ_j is interpreted as the imputed value of a delivered item, then (14) indicates that in the optimum case the cost of shipping one unit to destination j from warehouse i^*, t^*_{ij} must be greater than or equal to the imputed value of a delivered unit, λ_j.

It should be noted that λ_j represent the change in the minimum attainable cost resulting with the reduction of one unit in the quantity $[X\alpha_j \, \sigma r_j + \mu(r_j)]$ whose value

depends upon the selected values of α_j and X_{α_j} and the estimates of the parameters σr_j and μr_j, of the probability distribution.

Suppose that the manager of a shipping firm has signed a contract with the customer which demands that the shipping firm must meet the actual requirements of the customer 50 percent of the number times the order for shipment is placed. In this case $\alpha_j = .5$, and $X_{\alpha_j} = 0$. The constraint (13) would involve now only the expected values of the requirements distribution, $\mu(r_j)$. In short, the delivery firm is only concerned with the first moment of the requirement distribution. In this case λ_j measures the change in the minimum attainable cost associated with a 'small' change in $\mu(r_j)$.

If the shipping firm manager feels that the j-th customer's actual requirement must be met more than fifty percent of time, then α_j will be greater higher than 0.50 which implies that X_{α_j} will be greater than zero. For given σr_j $\mu(r_j)$ and α_j, the effective requirement for customer j is $[X_{\alpha_j} \sigma r_j + \mu(r_j)]$. The quantity $X_{\alpha_j} \sigma r_j$ can be thought of as an additional quantity that must be shipped (over expected requirement i.e. when $\alpha = .5$). A decrease in α_j can be determined so that

the value of $\sigma r_j \; X_{\alpha_j}$ is reduced by one unit. The effect of such a change in α_j on the minimum attainable cost is measured by λ_j. Let X_{α_j} correspond to the value of X_{α_j} which results in a unit reduction in the value of $[X_{\alpha_j} \; \sigma r_j + \mu(r_j)]$. Then $\alpha'_j - \alpha_j = \lambda_j$ where α'_j corresponds to X'_{α_j}.

3.4.3 Selection of α_j:

The value of α_j to be used in expression (8) is normally treated as a management decision parameter. The management can decide the value of α_j either using purely intuitive approaches or by coupling the intuitive approaches with the systematic procedure given in the following paragraphs.

The selection of the minimum confidence level should be based on a balancing of two different 'costs'. On one hand there is the cost of violating a constraint, i.e., of not meeting the minimum requirement at the destination. These 'costs' must be determined outside the model. In general, the higher the level of α_j selected the lower the expected level of these costs. On the other hand there are costs measured in terms of the objective function of the model which increase as the confidence levels increase since the problem then becomes more severely constrained.

The latter cost can be estimated using parametric programming. It would be worth while if both costs could be captured in the objective function by including a specific cost associated with the confidence level with which a constraint will be violated. In short, the determination of levels for α_j is properly apart of the optimizing model. This technique require a means of weighting the costs given in (9) and other costs associated with the selection of α_j. The weights must be determined through the use of a utility function for the firm's management.

If we let $\omega(\alpha_j)$ be the 'cost' associated with the violation of the j-th constraint then firm's utility function can be written as $U[Z, \omega(\alpha_1), \omega(\alpha_2) \ldots, \omega(\alpha_n)]$. If we fix each of the α_j's and minimize Z we have found one point on the efficiency frontier for this firm. The complete frontier can be obtained by parametrically altering the α_j's to find further best points on the frontier, of the firm.

The costs associated with not meeting a customer's minimum requirements will undoubtedly be difficult for the firm to measure since the loss incurred may not be purely monetary. Instead, the firm is more likely to be concerned with some estimates of frequency with which a customer's requirement will not be met even though it may not be able to

convert the frequency directly into cost. In other words, the manager of a delivery firm may not be able to estimate the cost of not satisfying the requirement of a particular customer. However, he may have some subjective feelings about how often, on the average, he can disappoint each customer without dire consequences. In short, the α_j parameter used in relation (6) may be taken to be the subjective estimate of this critical quality of service.

Besides, the utility function approach, the value of α_j can be determined by considering the value of number of units lost. Let UL_j represent the number of units lost for the j-th customer when the shipment is planned according to the desired α_j. Mathematically UL_j is given by,

$$\int_{d_j}^{\infty} (r_j - d_j)\, f(\phi)\, dr_j$$

Since, the demands are assumed to be normally distributed,

$$UL_j = \int_{d_j}^{\infty} (r_j - d_j) \frac{1}{\sqrt{2\pi}\sigma r_j} \exp - \frac{1}{2} \left(\frac{r_j - \mu(r_j)}{\sigma r_j}\right)^2 dr_j$$

$$= \int_{d_j}^{\infty} (r_j - \mu(r_j)) \frac{1}{\sqrt{2\pi}\sigma r_j} \exp - \frac{1}{2} \left(\frac{r_j - \mu(r_j)}{\sigma r_j}\right)^2 dr_j$$

$$+ \int_{d_j}^{\infty} (\mu(r_j) - d_j) \frac{1}{\sqrt{2\pi}\sigma r_j} \exp - \frac{1}{2} \left(\frac{r_j - \mu(r_j)}{\sigma r_j}\right)^2 dr_j$$

$$= \sigma r_j \frac{1}{\sqrt{2\pi}} \exp - \frac{1}{2} \left(\frac{d_j - \mu(r_j)}{\sigma r_j}\right)^2 + (\mu(r_j) - d_j)(1 - \alpha_j)$$

The management can convert the number of units lost into monetary value, ML_j, by assigning appropriate weight per unit lost for each customer,

$$ML_j = W_j \times UL_j$$

The value of ML_j decreases as α_j increases. On the other hand, the value of the objective function increases with increase in the value of α_j. Therefore, the optimum α_j corresponds to the point where the values of these two functions are equal.

3.5 Solution Methodology:

The problem as formulated in (9) to (13) is first solved as a linear-programming problem, ignoring the integer restriction on Y_i. If all the Y_i values obtained by solving the linear program are integers, then the problem has been solved. The solution procedure for solving the linear program is given below.

The assumption of unlimited warehouse capacity gives the linear program a structure which can be exploited to advantage as shown in Appendix I. Referring to Appendix I, we observe that,

$$X_{ij} = 1 \text{ if } C_{ij} + \frac{g_i}{n_i} = \min_{k \in K_1 \cup K_2} \left[C_{kj} + \frac{g_k}{n_k}\right] \qquad (20)$$

$$= 0 \text{ otherwise}$$

and,

$$Y_i = 0, \quad \text{for } i \in K_0$$

$$= \sum_{j \in P_i} X_{ij}/n_i , \qquad \text{for } i \in K_2$$

$$= 1, \text{ for } i \in K_1$$

where,

$$g_k = f_k, \quad \text{for } K \in K_2$$

$$= 0, \text{ for } K \in K_1$$

If for some K, Y_K is non-integer, then problem can be solved using number of methods available in the literature. Among these methods, the important ones are: enumerative techniques by Efroymson and Ray [4]. Khumawala [7,8] and Spielberg [18], group theoretic techniques by Kennington and Unger[10]; and heuristic procedures by Feldman, Lehrer and Ray [5] and Kuehn and Hamburger [12].

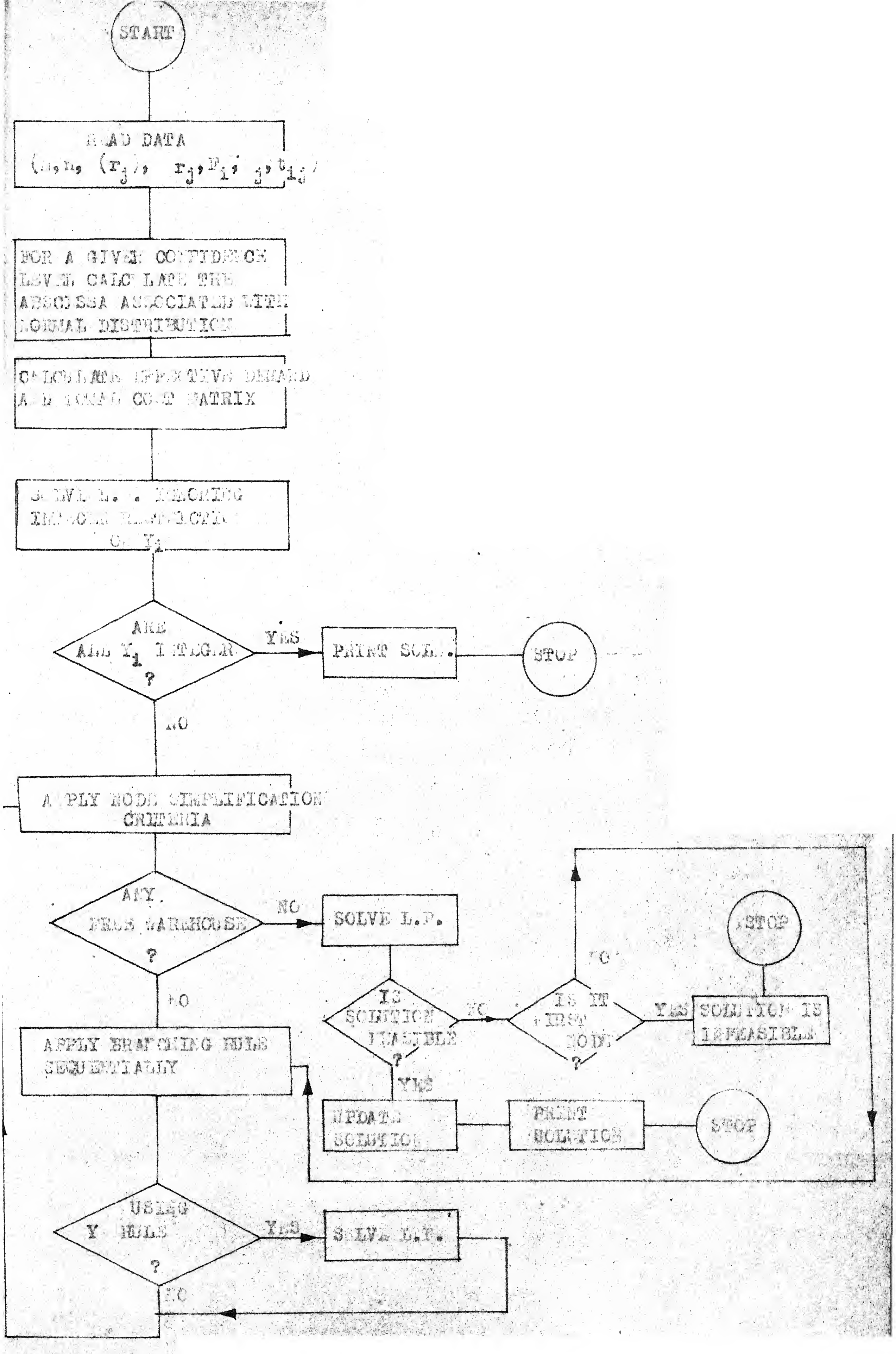
The problem of determination of the optimum number of warehouses selected for opening, out of the given m potential warehouses, is highly combinatorial in nature and the number of combinations which need to be considered increases as m increases. The total number of combinations involved would be $2^m - 1$. Due to combinatorial nature of the

problem, mostly computer oriented algorithms are developed to seek the optimal solution(s). The efficiency of a solution algorithm is generally characterized by the computational time and storage requirements. Khumawala [7] and Efrymson and Ray [4] claim that branch and bound procedures can be utilized efficiently for this class of problems. In the following sections the basic philosophy of branch and bound procedure is applied to the problem under investigation.

3.5.1 The Branch and Bound Algorithm:

The branch and bound procedure requires generating a sequence of partial assignments and analyzing the completions of each in search of successive better feasible solutions. There are many systematic ways of choosing a sequence of partial assignments that provides optimal or near optimal solution(s). The procedure we have employed can be represented by the flow diagram given in Fig. 5.

At the start of the algorithm, all warehouses are initialized as free and various node simplification criteria are applied. If at any node a free warehouse exists then branching rules are applied for fixed closing or opening of that warehouse. This procedure of branching and bounding continues till a node is reached where all Y_i



START
READ DATA
FOR A GIVEN CONFIDENCE LEVEL CALCULATE THE ABSCISSA ASSOCIATED WITH NORMAL DISTRIBUTION
CALCULATE EFFECTIVE DEMAND AND TOTAL COST MATRIX
ARE ALL Y_i INTEGER ?
YES
PRINT SOL.
STOP
NO
APPLY NODE SIMPLIFICATION CRITERIA
ANY FREE WAREHOUSE ?
NO
SOLVE L.P.
IS SOLUTION FEASIBLE ?
IS IT FIRST NODE ?
YES
SOLUTION IS INFEASIBLE
STOP
NO
APPLY BRANCHING RULE SEQUENTIALLY
YES
UPDATE SOLUTION
PRINT SOLUTION
STOP
USING Y RULE ?
YES
SOLVE L.P.
NO

values are integer. After reaching the node where all Y_1's are integer, a linear program is solved. The procedure is repeated with all branching rules and comparison done among the solutions obtained by various branching rules. The rule(s) which give(s) least objective function value is the optimal solution.

Further discussion on branch and bound algorithm requires familiarity with the definitions given in the next section.

3.5.1.1 <u>A Few Definitions</u>:

Terminal Node : A node where all Y_1 values are integer is called a terminal node.

Non-Terminal Node: A node which involves at least one fractional Y_1 value is known as non-terminal node.

Dangling Node: A node which may produce an improved solution is called a dangling node.

Closed Branch: The branch along which the warehouse is constrained closed is called closed branch.

Open Branch: The branch along which the warehouse is constrained open is called open branch.

3.5.1.2 Simplifications at Node: The following three node simplification criteria developed by Efroymson and Ray [4] have been used for reducing the number of branches at each node. These criteria are applied sequentially.

Criterion I: For Fixed Opening of Warehouse:

This node simplification criterion determines a minimum bound for cost reduction in 'opening' a warehouse. If this bound is positive, the warehouse is fixed 'open'.

Mathematically,

For $i \in K_2$ and $j \in P_i$

$$\nabla_{ij} = \underset{\substack{K \in N_j \cap (K_1 \cup K_2) \\ K \neq i}}{\mathrm{Min}} \left[\mathrm{Max}\,(C_{kj} - C_{ij},\ 0)\right] \tag{21}$$

$$\Delta_i = \sum_{i \in P_i} \nabla_{ij} - F_i \tag{22}$$

If $\Delta_i > 0$, then $Y_i = 1$ for all branches emanating from that node.

∇_{ij} measures the minimum cost saving for customer j that can be made if warehouse i is opened considering all the non-closed warehouses at that node. Clearly if the sum of such minimum savings for warehouse i over all customers that it can supply exceeds the fixed costs, F_i, it will always pay to open warehouse i at this node.

Criterion II. For Reduction of n_1:

This criterion reduces the total number of customers which can be supplied by a particular warehouse.

Mathematically,

For $i \in K_2$ and $j \in P_1$

If

$$\min_{K \in N_j \cap K_1} (C_{kj} - C_{ij}) < 0 \tag{23}$$

then n_1 is reduced by one. Of course, if the inequality holds for all $j \in P_1$, the $P_1 = \emptyset$. Therefore $P_1 = 0$ and $Y_1 = 0$ for all branches emanating from that node.

Clearly, if an already open warehouse can supply a customer j cheaper (in terms of lower variable costs) than any of the 'free' warehouses at that node, then such a customer should be supplied by the open warehouse. Therefore, such a customer should not be considered as a potential customer for the free warehouse at that node.

Criterion III: For fixed Closing of Warehouse:

This simplification determines a maximum bound on the cost reduction for opening a warehouse. If this bound is negative, the warehouse will be fixed closed. For

$i \in K_2$ and $j \in P_i$, the net saving, ω_{ij}, for the customer j when supplied from warehouse i can be expressed as,

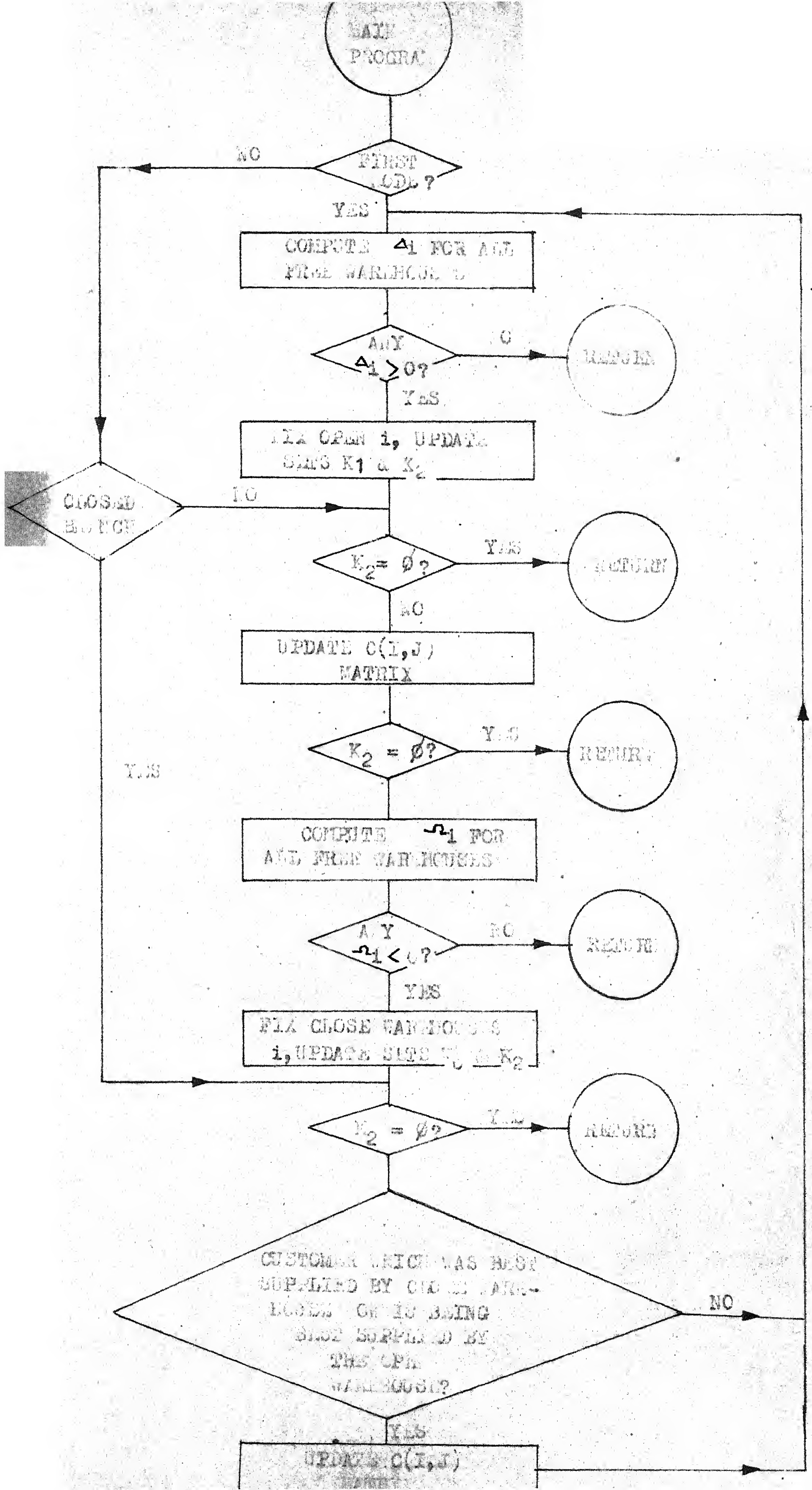
$$\omega_{ij} = \underset{K \in N_j \cap K_1}{\text{Min}} \left[\text{Max} \left(C_{kj} - C_{ij}, 0 \right) \right] \tag{24}$$

The total savings, Ω_i, which accrue from opening the warehouse i will be,

$$\Omega_i = \sum_{j \in P_i} \omega_{ij} - F_i \tag{25}$$

If $\Omega_i < 0$, then $Y_i = 0$ for all branches emanating from the node under consideration. It needs to be pointed out that ω_{ij} is similar to ∇_{ij} except that for ω_{ij} the comparisons are made only overall the fixed open warehouses and ω_{ij} is the minimum saving for supplying customer j from the opened warehouse i. Clearly, if the sum of such savings for warehouse i overall customers that it can supply fails to exceed its fixed cost F_i, then such a warehouse should be closed and eliminated from further consideration.

For each node these three node simplification criteria are applied sequentially till no further simplification is possible. It should be noted that if criterion I fails then Criterion II and III cannot be applied. The cycle through simplification is an important part of the procedure hence it is described by means of a separate flow chart given in Fig. 6.


PROGRA
NO
FIRST
YES
COMPUTE Δ_1 FOR ALL
ANY
$\Delta_1 > 0$?
YES
UPDATE
NO
YES
$K_2 = \emptyset$?
NO
UPDATE C(I,J)
MATRIX
$K_2 = \emptyset$?
RETURN
YES
COMPUTE Ω_1 FOR
ALL FREE WAREHOUSES
YES
FIX CLOSE
$K_2 = \emptyset$?
CUSTOMER
BEST
NO
YES
UPDATE C(I,J)

3.5.1.3 Branching Rules:

After applying all the simplification rules at a node if $K_2 \neq \emptyset$ then a warehouse is selected from the set of free warehouse at that node for further branching. The selected warehouse is either constrained closed or constrained open to yield an additional node. Instead of selecting the warehouse randomly, the following decision rules developed by Khumawala [8] are used to arrive at an appropriate node for consideration.

Delta Rules:

In the node simplification criterion I, Δ_i's are computed for each free warehouse at every node. If $\Delta_i \geq 0$ then warehouse i is fixed open for all branches emanating from that node. However, for the warehouses for which Δ's are negative no decision is taken. Warehouses having large (i.e. small negative) Δ values are likely to be opened in the terminal solution from this node. On the other hand, those warehouses having small (i.e. high negative) Δ values are likely to be closed in the terminal solution from this node. We have two branching rules based on 's.

(1) Largest Delta Rule: Select the free warehouse for fixed opening which has the largest Δ from the set of free warehouses.

(2) Smallest Delta Rule: Select the free warehouse for fixed closing which has the smallest Δ from the set of free warehouses.

Omega Rules:

In the node simplification criterion III, Ω_i's are computed for each warehouses at each node. If $\Omega_i \leq 0$ then warehouse i is fixed closed for all branches emanating from that node. However, for the warehouses whose Ω are positive no decision is taken. Therefore warehouses having largest positive Ω values are likely to be open in the terminal solution reached from this node and vice versa. Two branching rules based on considerations are:

(1) Largest Omega Rule: Select the free warehouse for fixed opening which has largest positive Ω from the set of free warehouses.

(2) Smallest Omega Rule: Select the free warehouse for fixed closing which has the smallest positive Ω from the set of free warehouses.

Y Rules: For applying these rules a linear programming problem is solved for the selected node. Whether a warehouse is fixed open, fixed closed or free depends on the value of Y_i. If $Y_i = 1$, the warehouse is fixed open, if $Y_i = 0$, the warehouse is fixed closed and if Y_i is fractional, the

warehouse is free. A free warehouse whose Y is close to one is more likely to be open in the terminal solution reached from that node as compared to the warehouses whose Y is less. Conversely the warehouse whose Y is close to zero is likely to be closed in the terminal solution reached from the node. This leads to two branching rules based on the Y's.

(1) Largest Y Rule: Select the free warehouse for fixed opening with the largest Y from the set of free warehouses at the node having fractional Y.

(2) Smallest Y Rule: Select the free warehouse for fixed closing with the smallest Y from the set of free warehouses at the node having fractional Y.

Demand Rule: The rationale here is that if a warehouse is supplying a large demand currently it is likely to be open in the terminal solution reached from the node and vice versa. The two branching rules based on demand considerations are:

(1) Largest Demand Rule: Select the free warehouse for fixed opening which is supplying the largest demand from free warehouses at that node.

(2) Smallest Demand Rule: Select the free warehouse for fixed closing which is supplying the smallest demand from free warehouses at that node.

3.5.1.4 Use of Branching Rules for Node Selection.

The use of the branch and bound procedure involves large computer storage and high computational time. This is because the number of nodes that need to be evaluated to reach an optimal solution has an exponential relationship with the total number of integer variables in the problem. For reducing the number of nodes which need to be considered for evaluation, node selection procedures developed by Khumawala [8] have been used. The node selection procedure is used in conjunction with each branching rule. In essence, the node selection procedure can be stated as, 'Trace the preferred path which would result from an application of the particular branching decision rule'.

The word 'preference' is used here to mean the motivation on which the free warehouse is selected. As the branching rule is applied in the branch and bound algorithm from the initial node [node 1, Fig. 7] we expect the best solution to be along some preferred branch. If the node resulting from the preferred branch is not terminal branching will continue along the preferred path till a terminal node is reached. In figure 7, the preferred path will be 1-3-5-7- n.

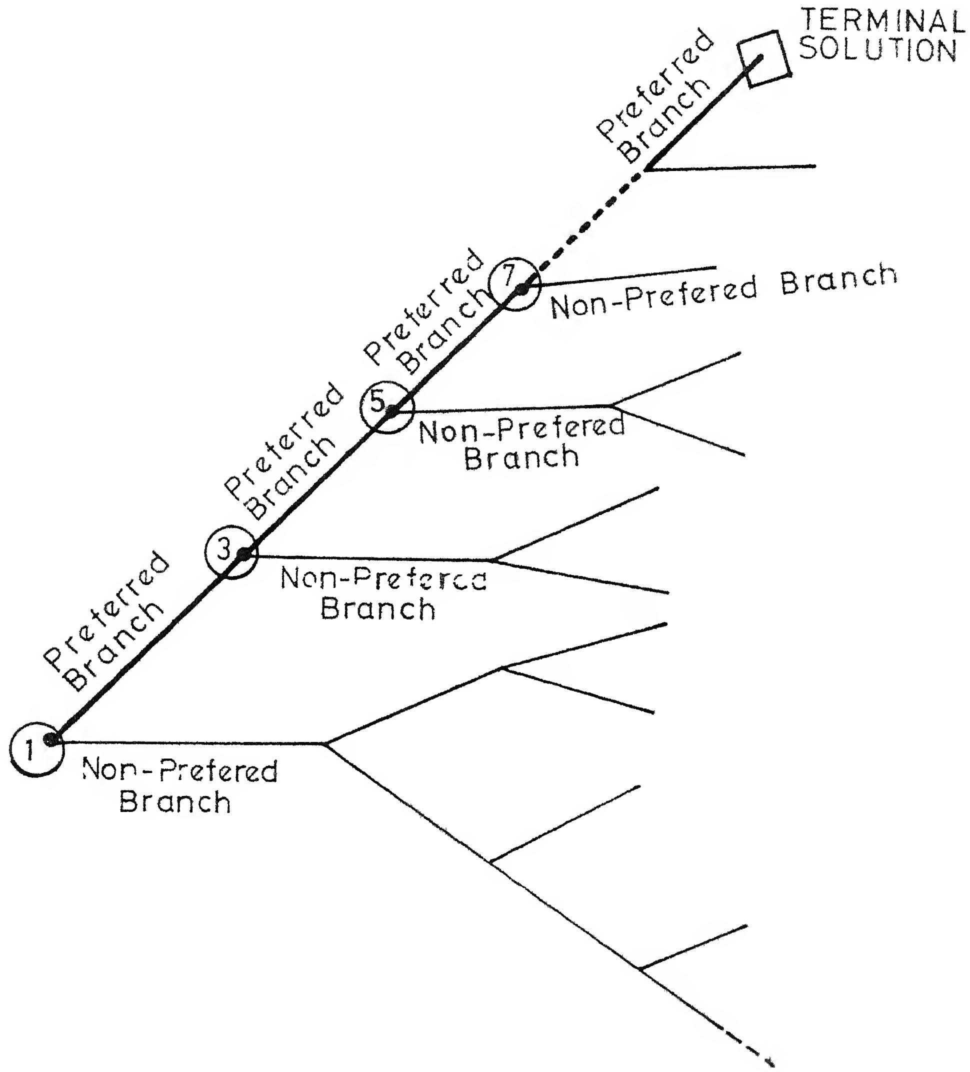


FIG. 7 THE PREFERRED PATH.

Illustrative Example I: (For Selection of α_j)

A numerical example is considered here to illustrate the methodology for selection of α_j.

Conforming to the notation, the input data is:

$m = 10$, $n = 15$, $\mu(r_j) = 26.00$ for $\forall_j$ and ~~$\sigma r_j = 5.3$~~
and $\sigma r_j = 5.3$ for all $\forall_j$

The fixed costs for the various warehouses (1,2,...10) are (60, 185, 95, 65, 120, 70, 150, 90, 50, 80) respectively.

The values of (t_{ij}) are given in Table 3.1. It is assumed that for all columns $\alpha_j = \alpha$ a given constant. The various α's considered are .5, .6, .7, .8, .9 and .95.

For various values of α, the values of satisfied demand d_j, the value of units lost (UL_j), the value of the objective function are presented in Table 3.2. Fig. 8 is a graphic representation of total cost and number of units lost for various values of α. By assigning appropriate weightages to the unit lost, the unit lost curve can be converted into a cost curve. The super imposition of these two cost curves will result in a curve with a unique minima. The optimal value of α will correspond to the point where the super-imposed cost curve has a minimum value.

Table 3.1: Unit Cost From Warehouse i to Customer j

Warehouses	Customers 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	210	215	236	307	350	109	136	142	276	207	240	199	105	250	218
2	207	218	203	311	355	86	112	144	257	209	270	207	110	246	207
3	236	217	205	316	360	86	187	156	242	215	100	204	400	237	190
4	215	209	219	315	367	215	158	152	271	205	400	308	298	187	187
5	204	225	216	307	347	218	177	170	207	200	350	311	350	157	194
6	227	217	300	350	358	107	120	240	262	211	280	315	247	176	237
7	237	206	308	304	307	220	115	241	203	100	390	251	150	187	276
8	240	208	320	318	319	176	112	247	244	107	157	215	156	170	208
9	230	228	329	315	327	127	154	249	256	108	258	272	176	190	210
10	207	234	310	360	370	115	147	258	254	110	198	376	201	245	207

Table 3.2

α	d_J	UL_j	Z
.5	26.00	31.80	18900
.6	27.3430	21.34050	19892
.7	28.7804	14.93820	20921
.8	30.4621	8.96370	22125
.9	32.9751	3.21735	23796
.95	34.7227	3.042975	25176

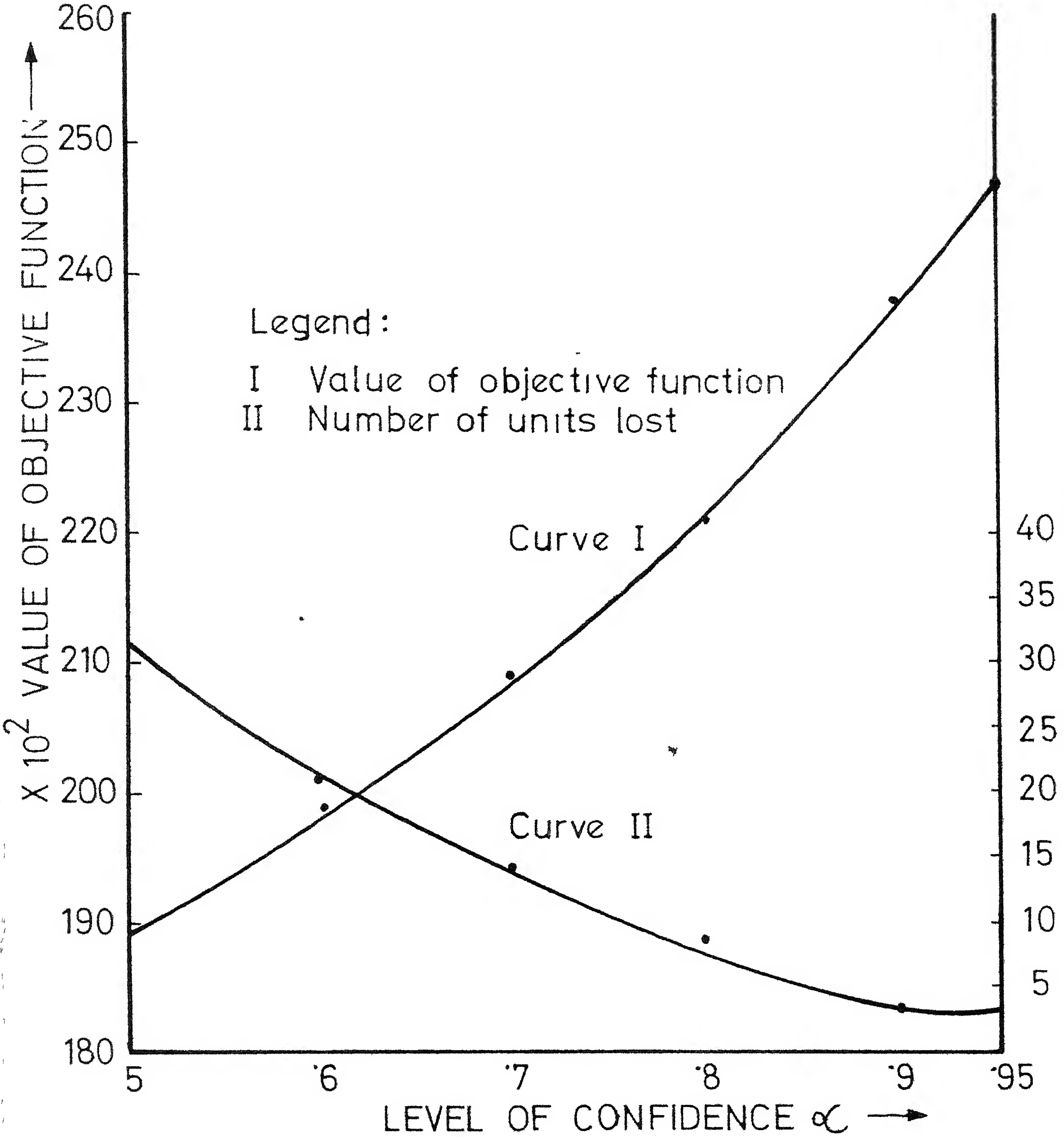

260
250
240
230
220
210
200
190
180
X 10^2 VALUE OF OBJECTIVE FUNCTION
Legend:
I Value of objective function
II Number of units lost
Curve I
Curve II
40
35
30
25
20
15
10
5
NUMBER OF UNITS
5
·6
·7
·8
·9
·95
LEVEL OF CONFIDENCE α

FIG. 8

Illustrative Example II. (For the Procedure)

A problem of (4 x 6) has been structured and solved to illustrate the various steps. The input data for the problem is given in Tables 3.3 and 3.4.

Table 3.3

Customers	1	2	3	4	5	6
Desired confidence level for meeting the demand α_j	.8	.87	.9	.85	.98	.95
Mean Demand $\mu(r_j)$	12.0	5.0	16.0	10.0	25.0	30.0
Standard Deviation σ_{r_j}	.4	.01	.1	.54	.8	2.1

Table 3.4: Unit Cost Matrix (t_{ij}) and Fixed Cost F_i

Ware-house	Fixed Cost	Customers 1	2	3	4	5	6
1	84	1.2	9.0	7.0	9.0	14.0	4.5
2	60	3.0	8.0	6.0	16.0	12.0	12.0
3	120	1.1	10.0	8.0	10.0	15.0	15.0
4	72	1.3	12.0	9.0	7.0	11.0	8.0

Step I: Find $X_{\alpha j}$, i.e, the abscissa associated with left tail of the standard normal distribution function for the confidence level α_j. Table 3.5 represents the effective

demand of each customer which is obtained by the expression.
$D_j = X_{\alpha j}\ \sigma r_j + \mu(r_j)$.

Table 3.5

Customers	1	2	3	4	5	6
$X_{\alpha j}$	.8418	.1126	.1282	.1036	.2055	.1645
D(j)	12.34	5.01	16.12	10.55	26.64	33.45

Step II Obtain (C_{ij}) matrix by multiplying demand of j-th customer with corresponding column of t_{ij} matrix.

Table 3.6: C_{ij} Matrix

	Customer					
Warehouse	1	2	3	4	5	6
1	14.80	45.10	112.89	95.03	373.02	150.55
2	37.01	40.09	96.76	163.95	319.73	410.74
3	13.57	50.11	129.02	105.59	399.66	501.84
4	16.03	60.13	145.15	37.90	293.09	267.34

Step III: Solve the problem as linear programming problem without integer restriction on Y_i. From relation (20) the value of $(C_{ij} + (g_i/n_i))$ are calculated and given in Table 3.7. If all the Y_i are integer then problem is solved otherwise go to Step IV.

Table 3.7: $C_{ij} + \frac{g_i}{n_i}$

Warehouse	Customers 1	2	3	4	5	6
1	28.80	59.10	127.89	109.03	387.02	164.55
2	47.01	50.09	106.76	178.95	329.73	420.74
3	33.50	70.11	149.02	125.59	419.66	521.84
4	28.03	72.13	157.15	85.09	305.09	279.64

Therefore $X_{14} = X_{22} = X_{23} = X_{41} = X_{44} = X_{45} = 1$ and all other $X_{ij} = 0$ and $Y_1 = 1/6$, $Y_2 = 2/6$, $Y_3 = 0/6$ and $Y_4 = 3/6$.

Since all Y_i are not integer therefore procedure will continue.

Step IV: Initialize the node and all warehouses as free.

Step V: If it is the first node go to Step VI, if any other and the branch is open go to Step VIII otherwise to Step XII.

Step VI: Calculate ∇_{ij} and Δ_i. Check if any Δ_i is positive, open that warehouse otherwise go to Step XIV.

$\nabla_{11} = 0, \nabla_{12} = 0, \nabla_{13} = 0, \nabla_{14} = 0, \nabla_{15} = 0, \nabla_{16} = 0, \nabla_{16} = 117.$

$\nabla_{21} = 0, \nabla_{22} = 5.01, \nabla_{23} = 16.13, \nabla_{24} = 0, \nabla_{25} = 0, \nabla_{26} = 0$

$\nabla_{31} = 1.23, \nabla_{32} = 0, \nabla_{33} = 0, \nabla_{34} = 0, \nabla_{35} = 0, \nabla_{36} = 0$

$\nabla_{41} = 0, \nabla_{42} = 0, \nabla_{43} = 0, \nabla_{44} = 21.07, \nabla_{45} = 26.64, \nabla_{46} = 0.$

Now,

$$\Delta_1 = 117.09 - 84 = 33.09$$
$$\Delta_2 = 16.13 + 5.01 - 60 = -38.86$$
$$\Delta_3 = 1.23 - 120 = -113.76$$
$$\Delta_4 = 21.07 + 26.61 - 72 = -24.29$$

Because $\Delta_1 > 0$, hence warehouse 1 is fixed open, therefore $K_1 = [1]$ and $K_2 = [2,3,4]$.

Step VII: If $K_2 = [\emptyset]$ go to Step XVII.

Step VIII: If a particular customer is best supplied by an already fixed open warehouse it is removed from further consideration. Here customer 6 is being supplied the cheapest.

Table 3.8

Warehouse	Customers 1	2	3	4	5	6
1	14.80	45.10	112.89	95.03	373.02	150.55
2	37.01	40.09	96.76	168.95	319.73	-
3	13.57	50.11	129.02	105.59	399.76	-
4	16.03	60.13	145.15	73.90	293.09	-

Step IX: Update K_2, if $K_2 = [\emptyset]$, go to Step XVII.

Step X : Calculate ω_{ij} and Δ_i for free warehouses. If any Δ_i is negative, close that warehouse otherwise go to Step XIV.

$\omega_{21} = 0$, $\omega_{22}=5.01$, $\omega_{23}=16.13$, $\omega_{24}=0$, $\omega_{25}=53.29$

$\omega_{31}=1.23$, $\omega_{32}=0$, $\omega_{33} = 0$, $\omega_{34}=0$, $\omega_{35} = 0$

$\omega_{41} = 0$, $\omega_{42}=0$, $\omega_{43} = 0$, $\omega_{44}=21.13$, $\omega_{45} = 79.93$

Now,

$$-\lambda_2 = 5.01 + 16.13 + 53.29 - 60 = 14.43$$

$$-\lambda_3 = 1.23 - 120 = -118.77$$

$$-\lambda_4 = 21.13 + 79.93 - 72 = 19.06$$

Here $-\lambda_3 < 0$ hence it will befixed close and thus $K_0 = [3]$, $K_1 = [1]$, $K_2 = [2, 4]$.

Step XI: If $K_2 = [\emptyset]$, go to Step XVII.

Step XII: Delete the customer which can be best supplied from an open warehouse forthe further consideration for free warehouse. Here customer 1 is being the best supplied after closing of warehouses 3.

Table 3.9

Warehouse	Customer 1	2	3	4	5	6
1	14.80	45.10	112.89	95.03	373.00	150.55
2	-	40.09	96.76	168.95	319.73	-
3	-	-	-	-	-	-
4	-	60.13	145.15	73.90	293.09	-

Step XIII: Go to Step IV.

Step XIV: If $K_2 = [\emptyset]$ goto step XVII.

Step XV: Select the warehouse according to the branching rule that is being applied, i.e. if largest delta rule is being applied then warehouse 4 will be fixed open since Δ_4 is the largest among all Δ's.

$$\Delta_2 = -38.86$$

$$\Delta_4 = -24.23$$

Step XVI: Go to Step V.

Step XVII: Solve linear programming and we find

$$X_{11} = 1, \; X_{12} = 1, \; X_{13} = 1, \; X_{16} = 1, \; X_{44} = 1, \; X_{45} = 1,$$

all other $X_{ij} = 0$ and value of objective function is 846.3650.

Step XVIII: Go to Step IV until all branching rules are exhausted.

Step XIX: Compare various solutions obtained by the different branching rules and obtain minimum value.

CHAPTER IV

RESULTS AND DISCUSSIONS

A computer program of the methodology proposed in Chapter III has been developed. The program is written in Fortran IV. A listing of the program is given in Appendix III. The input data requires the values of m, n, r_j, $u(r_j)$, $\sigma(r_j)$, t_{ij}, F_i. For the validation of the procedure a problem from literature given by Khumawala [8] has been selected. Since this problem assumes deterministic customers' demand, a value of α = .5 has been used for all the customers. The results obtained using the proposed methodology are found to be the same as those of Khumawala. For evaluating the computational efficiency and limitations of the solution methodology, twenty problems were generated and solved. In all, problems of 4 different sizes, viz. (10x15), (15x20), (20x25) and (25x50) have been considered. The first number corresponds to the number of warehouses and the second number represents the number of customers. The computational time requirement for a problem of given size has been determined based on the average time required for solving a set of 5 problems of the same size. In Table 4.1, computational

time requirements for problems of various sizes are tabulated. In Table 4.2, the average times for using the various branching rules is given. The performance of various branching rules to generate optimal solutions is tabulated in Table 4.3.

Table 4.1: Average Computation Time Required For Problems of Different Sizes.

No. of Warehouse	No. of Customers	Average time in secs.
10	15	20
15	20	41
20	25	44
25	50	55

Table 4.2: Computational Time Requirements for Various Branching Rules.

Size of Problem	Branching Rules							
	LDR	LOR	LYR	LDR	SDR	SOR	SYR	SLR
10 x 15	.2	.4	.6	.2	.2	.4	.6	.2
15 x 20	1.0	1.2	1.5	1.0	1.0	1.3	1.6	1.0
20 x 25	1.5	1.7	2.0	1.9	1.7	1.0	1.8	1.0
25 x 50	6	7	12	6	6	8	20	6

Table 4.3: Performance Evaluation of Various Branching Rules.
(Frequency of Obtaining Optimal Solution with Various Branching Rules out of 5)

Problem size	LDR	LOR	LYR	LDR	SDR	SOR	SYR	SDR
10 x 15	5	5	4	5	5	5	5	5
15 x 20	5	5	4	5	5	5	5	4
20 x 25	5	5	4	4	4	5	4	4
25 x 50	5	5	2	3	4	3	2	2

The following important inferences have been drawn from the results.

(1) Regardless of the branching rule applied, the computer times required for solving the 4 different sizes of problems considered, is very low (Refer Table 4.1). Therefore, the procedure can be used for fairly large realistic warehouse location problems with reasonable computer time.

(2) It is observed that for problems of smaller size, e.g., (10 x 15, 15 x 20), all branching rules give the optimal solution but for large size of problems e.g. (20 x 25, 25 x 50), Largest Delta and Largest Omega rules generally give the optimal solution.

(3) Y rules always take more time. This can be attributed to the fact that application of these rules need solving of linear programming.

(4) Since Largest Delta and Largest Omega rules generally give optimal solution, it is, therefore, recommended that whenever one is reluctant to apply all these rules to arrive at the optimal solution, these two rules should be applied. Y rules should be given least priority since they always take more computational time.

4.1 <u>Suggestions for Future Work</u>:

In the beginning, whenever a real life situation is modelled the model has to be, of necessity, kept simple either because the actual constraints cannot be included in a simple way or because solving the model becomes intractable. To mirror real life processes the model can be improved and extended in many ways. Below we mention a few directions in which further work can be done towards this end.

1. The model presented in this thesis is based on the assumption that the customer's demand is normally distributed. Instead, the demand could follow other general distributions like Poisson, Binomial and Exponential distributions. It would be interesting to develop models and the solution methodologies for situations when all the customer's demand don't follow the same distribution.

2. In the present model the α_j's are being determined outside the model. However, a model needs to be developed where the determination of optimal α_j is treated an integral part of the model.

3. It is assumed that warehouses are uncapacitated. However, in real life situation, there is always a limit on the size of the warehouse. Therefore, models for the location of capacitated warehouses considering probabilistic demands need to be developed.

4. For an exercise like this to be more meaningful, the system must be considered as a whole. Goods are rarely transported directly from plant sites to customers, that is to say that warehouses are not located at the plant. Normally goods are shipped from the factory to the warehouses and thence onward to the customers. This two tier transportation model should be included in the formulation for a more realistic approach.

5. The model needs to be further extended for nonlinear transportation costs and constraints like budget, etc.

6. An implicit assumption in this work is that we are working in a discrete space. However, it would be worth-while to develop models for locating warehouses in continuous space.

7. In this thesis, eight heuristic branching rules have been used and evaluated. However, one could conceive of many more branching rules. A few more rules which can be tested are:

Largest n_i Rule: At a given node, from the set of free warehouses, select the warehouse which is supplying largest number of customers for fixed opening.

Smallest n_i rule: At a given node, from the set of free warehouses, select the warehouse which is supplying smallest number of customers for fixed closing.

Largest Average Cost Rule: At a given node, from the set of free warehouses, select the warehouse which is supplying at the largest average cost ($\sum_{j=1}^{n}(C_{ij}+F_i)/n_i$) for fixed closing.

Smallest Average Cost Rule: At a given node, from the set of free warehouse, select the warehouse which is supplying at the smallest average cost ($\sum_{j=1}^{n}(C_{ij}+F_i)/n_i$) for fixed opening.

8. The node simplification procedure used is based on simple logic. However, some mathematical approaches for node simplification need to be developed. Hopefully, a more sophisticated node simplification procedure will reduce the computational effort.

REFERENCES

1. Charnes, A., and Cooper, W.W., 'Chance-Constrained Programming', Management Science, Vol. 6, (1959), pp. 73-80.

2. Cooper, L., 'Location-Allocation Problems', Operations Research, Vol. 11 (1963), pp. 331-343.

3. Drysdale, J.K., and Sandiford, P.J.,'Heuristic Warehouse Location - A Case History Using a New Method', Canadian Operational Research Journal, Vol. 7 (1969), pp. 45-61.

4. Efroymson, M., and Ray, T., 'A Branch and Bound Algorithm for Plant Location', Operations Research, Vol. 14 (1966), pp. 361-368.

5. Feldman, E., Lehrer, F., and Ray, T., 'Warehouse Location Under Continuous Economies of Scale', Management Science, Vol. 12 (1966), pp. 670-684.

6. Francis, R.L., and White, J.A., 'Facility Layout and Location: An Analytical Approach', Prentice Hall. 1974.

7. Khumawala, B.M., 'An Efficient Branch and Bound Algorithm for Warehouse Location', Management Science, Vol. 18 (1972), pp. 718-731.

8. Khumawala, B.M., 'An Efficient Heuristic Procedure for the Uncapacitated Warehouse Location Problem', Naval Research Logistics Quarterly, Vol. 20 (1973).pp.109-21.

9. Khumawala, B.M., and Whybark, D.C., 'An Update on Warehouse Location Problem', The Logistics and Transportation Review, Vol.9 (1973).

10. Kennington, J. and Unger, V., 'The Group Theoretic Structure in the Fixed Charge Transportation Problem', Management Science, Vol. 8 (1972) pp.718-731.

11. Kreyszig, E., 'Introductory Mathematical Statistics,' Wiley International Edition, 1970.

12. Kuehn, A., and Hamburger, N., 'A Heuristic Approach for Locating Warehouse', Management Science, Vol. 10 (1963), pp. 643-666.

13. Little, J.D.C., Murthy,K.G., Swenny, D.W., and Karel, C.,'An Algorithm for the Travelling Salesman Problem', Operations Research, Vol. 11, pp. 972-989.

14. Magee, J.F., 'Industrial Logistics, Analysis and Management of Physical Supply and Distribution Systems, McGraw-Hill, 1967.

15. Markland, R.E., 'Analyzing Geographically Discrete Warehousing Networks by Computer Simulation', Decision Sciences, Vol. 4 (1973), pp. 216-236.

16. Marks, D.H., 'Facility Location and Routing Models in Solid Waste Collection Systems', Ph.D. Thesis, The Johns Hopkins University, November, 1969.

17. Revelle, C., Marks, D., Liebman, J.C., 'An Analysis of Private and Public Sector Location Models', Management Science, Vol. 16 (1970), pp. 691-707.

18. Spielberg, K., 'Algorithms for the Simple Plant Location Problem with Some Side Conditions', Operations Research, Vol. 17 (1969), pp. 85-111.

19. Shycon, H.N., and Maffei, R.B., 'Simulation-Tool for Better Distribution', Harvard Business Review, Vol. (1960), pp. 65-75

20. Walker, W., and Lynn,W., Adjacent Extreme Point Algorithm for Fixed Charge Problem', Paper submitted to Management Science.

APPENDIX I

SIMPLIFICATION OF LINEAR PROGRAM

The assumption of unlimited warehouse capacity gives the linear program a structure which can be exploited advantage as shown below:

We have,

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} F_i Y_i \qquad (1)$$

$$\text{S.t.} \quad \sum_{i=1}^{m} X_{ij} = 1 \qquad j = 1, 2, \ldots, n \qquad (2)$$

$$\sum_{j=1}^{n} X_{ij} \leq nY_i \qquad i = 1, 2, \ldots, m \qquad (3)$$

$$X_{ij} \geq 0 \qquad \forall i \text{ and } j \qquad (4)$$

$$Y_i = (0, 1) \qquad \forall i \qquad (5)$$

Let K_0, K_1 and K_2 represent the sets of warehouses which correspond to fixed close, fixed open and free warehouse, then we have,

$$K_0 = [i : Y_i = 0]$$

$$K_1 = [i : Y_i = 1]$$

$$K_2 = [i : Y_i \text{ unassigned}]$$

and,

$$K_0 \cup K_1 \cup K_2 = [1,2,\ldots, n]$$

The linear programming problem can now be stated as:

$$\text{Minimize} \quad Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i=1}^{m} F_i Y_i \tag{6}$$

$$\text{S.t.} \quad \sum_{i=1}^{m} X_{ij} = 1 \qquad j = 1,2,\ldots, n \tag{7}$$

$$\sum_{j=1}^{n} X_{ij} \leq 0 \qquad i \in K_0 \tag{8}$$

$$\sum_{j=1}^{n} X_{ij} \leq n \qquad i \in K_1 \tag{9}$$

$$\sum_{j=1}^{n} X_{ij} \leq nY_i \qquad i \in K_2 \tag{10}$$

$$X_{ij} \geq 0 \qquad \text{for } \forall\ i \text{ and } j \tag{11}$$

$$Y_i \geq 0 \qquad i \in K_2 \tag{12}$$

Since $X_{ij} \geq 0$, from (8) we get $X_{ij} = 0$ for all j belonging to the set K_0.

Furthermore, using (7), we obtain,

$$\sum_{j=1}^{n} \sum_{i=1}^{m} X_{ij} = \sum_{j=1=}^{n} 1 = n$$

Since,

$$\sum_{j=1}^{n} X_{ij} \leq \sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} = \sum_{j=1}^{n} 1 = n$$

the constraint (9) is redundant.

Referring to expression (6), the first term,

$$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} X_{ij} = \sum_{i \varepsilon K_1 U K_2} \sum_{j=1}^{n} C_{ij} X_{ij}$$

Since,

$$X_{ij} = 0 \quad \text{for} \quad j \varepsilon K_0$$

and

$$\sum_{i=1}^{m} X_{ij} = \sum_{i \varepsilon K_1 U K_2} X_{ij}$$

Since $F_i = 0$ for $i \varepsilon K_0$, the second term of expression (6) can be written as,

$$\sum_{i=1}^{m} F_i Y_i = \sum_{i \varepsilon K_1} F_i + \sum_{i \varepsilon K_2} F_i Y_i$$

Now the problem given by (6) to (12) can be stated as:

$$\text{Minimize} \quad \hat{Z} = \sum_{i\varepsilon K_1 UK_2} \sum_{j=1}^{n} C_{ij} X_{ij} + \sum_{i\varepsilon K_1 UK_2} F_i Y_i \qquad (13)$$

$$\text{S.t.} \quad \sum_{i\varepsilon K_1 UK_2} X_{ij} = 1 \qquad i = 1, 2, \ldots, m \qquad (14)$$

$$\sum_{j=1}^{n} X_{ij} - nY_i \qquad i \varepsilon K_2 \qquad (15)$$

$$X_{ij} \geq 0 \qquad i \varepsilon K_1 UK_2$$
$$j = 1, 2, \ldots, n \qquad (16)$$

$$Y_i \geq 0 \qquad i \varepsilon K_2 \qquad (17)$$

where,

$$Z = \hat{Z} + \sum_{i\varepsilon K_1} F_i$$

The above stated problem can be further simplified by using the following axioms.

<u>Axiom 1</u>:

Let X^*_{ij} and Y^*_i represent the minimum feasible solution for the problem given in (13) to (17) and if all $F_i > 0$ then,

$$\sum_{j=1}^{n} X_{ij} = nY_i \qquad i \varepsilon K_2 \qquad (18)$$

$$Y_i \leq 1 \qquad i \varepsilon K_2 \qquad (19)$$

Proof:

To prove (18), notice if for some $i \varepsilon K_2$

$\sum_{j=1}^{n} X_{ij} < nY_i$, then by letting Y_i equal to

$\frac{1}{n} \sum_{j=1}^{n} X_{ij}$ the value of Z can be reduced.

Consequently for X_{ij}^* and Y_i^* to be optimum (18) must hold. To prove (19) we observe that,

$$Y_i = \frac{1}{n} \sum_{j=1}^{n} X_{ij} \leq \frac{1}{n} \sum_{j=1}^{n} \sum_{i \varepsilon K_1 U K_2} X_{ij} = \frac{1}{n} \sum_{j=1}^{n} 1 = 1$$

Applying Axiom 1, problem (13) to 17) can be reduced to,

$$\text{Minimize} \quad \hat{Z} = \sum_{j=1}^{n} \sum_{i \varepsilon K_1 U K_2} C_{ij} X_{ij} + \sum_{i \varepsilon K_2} F_i Y_i \qquad (20)$$

$$\text{S.t.} \quad \sum_{i \varepsilon K_1 U K_2} X_{ij} = 1 \qquad j = 1, 2, \ldots, n \qquad (21)$$

$$\sum_{j=1}^{n} X_{ij} = n Y_i \qquad i \varepsilon K_2 \qquad (22)$$

$$X_{ij} \geq 0 \qquad i \varepsilon K_1 U K_2, \quad j = 1, 2, \ldots, n \qquad (23)$$

$$Y_i \geq 0 \qquad i \varepsilon K_2 \qquad (24)$$

A further simplification of the problem can be achieved by making use of following axiom.

<u>Axiom 2</u>:

Define a new term g_i where

$$g_i = \begin{cases} 0 & \text{if } i \in K_1 \\ f_i & \phantom{\text{if }} i \in K_2 \end{cases}$$

Given any feasible solution X_{ij}, Y_i to problem represented by (20) to (24), then

$$\sum_{i \in K_2} f_i Y_i = \sum_{j=1}^{n} \sum_{i \in K_1 \cup K_2} \frac{g_i}{n} X_{ij}$$

<u>Proof</u>:

Let X_{ij} and Y_i be a feasible solution to the problem given by (20) to (24), then from (22),

$$\sum_{i \in K_2} f_i Y_i = \sum_{i \in K_2} F_i \left(\frac{1}{n} \sum_{j=1}^{n} X_{ij} \right)$$

$$= \sum_{i \in K_1 \cup K_2} \frac{g_i}{n} \sum_{j=1}^{n} X_{ij}$$

$$= \sum_{i \in K_1 \cup K_2} \sum_{j=1}^{n} \frac{g_i}{n} X_{ij}$$

Thus the problem can now be stated as,

$$\text{Minimize} \quad Z = \sum_{j=1}^{n} \sum_{i \varepsilon K_1 U K_2} \left(C_{ij} + \frac{g_i}{n}\right) X_{ij} \qquad (25)$$

$$\text{S.t.} \quad \sum_{i \varepsilon K_1 U K_2} X_{ij} = 1 \text{ , } \quad j = 1, 2, \ldots, n \qquad (26)$$

$$X_{ij} \gtrless 0 \qquad i \varepsilon K_1 \; U K_2 \qquad (27)$$

$$j = 1, 2, \ldots, n \qquad ($$

It is interesting to note that problem given in (25) to (27) is essentially an assignment problem. Thus, we have

$$\lambda_{ij} = 1 \quad \text{if } C_{ij} + \frac{g_i}{n_i} \quad \underset{K \varepsilon K_1 U K_2}{\text{Min}} \quad \left(C_{kj} + \frac{g_i}{n_i}\right)$$

$$= 0$$

From the definition of K_0 and K_1, and using expression (22), we have,

$$Y_i = 0 \qquad \text{for } i \varepsilon K_0$$

$$= \sum_{j=1}^{n} X_{ij}/n \qquad \text{for } i \varepsilon K_2$$

$$= 1 \qquad \text{for } i \varepsilon K_1$$

APPENDIX II

INFORMATION FOR USERS' OF THE PROGRAM

This program implements the branch and bound algorithm for solving the uncapacited warehouse location problem under probabilistic demand. The program utilizes eight heuristic rules for node selection. A discussion of the rules and their efficacy in arriving at a proper solution is given in the main body of the text of this thesis.

The data structures used are one and two dimensional arrays. The status of warehouses is represented by three integers -1, 0, 1 which stand for closed, free and open warehouses respectively. The program in its present form can be used to solve problems involving a maximum of 50 warehouses and 100 customers. However, every care has been taken to make the program comprehensive. The program is execution time dimensioned and can be used to solve a problem of any size depending on core memory availability simply by changing the dimension statements in the main program.

Input Preparation:

Card No.	Format	Symbol	Description
1	I_2	NPROB	Number of problems
2	I_3, I_4	M, N	M : No. of warehouses N : No. of customers
3	10 F.4.2	P(J)	Minimum level of confidence with which j-th customer's actual demand must be satisfied
4	10 F 7.3	AMU(J)	Mean of random variable R(J)
5	10 F 7.3	SIGMA(J)	Standard deviation of the random variable R(J)
6	10 F 6.2	F(I)	Fixed charge resulting from establishing a warehouse at site I
7	10 E 7.1	C(I,J)	Unit transportation cost from I-th warehouse to J-th customer. When route is prohibited put C(I,J) = 1.0E 30 is same format

Output:

For each problem the program outputs the following quantities:

1. The status of warehouses (i.e. whether open, close or free) for each branching rule.
2. The set of customers supplied by each warehouses.
3. Value of the objective function for each branching rule.
4. Execution time for the application of each branching rule (this is done by calling library subroutine TIME).
5. The minimum value of objective function from the set of value in (3) above.
6. The rules which give minimum objective function value.

The above is a minimum output configuration. If a detailed knowledge of the status of the warehouses after each simplification is required or if the changes in the cost matrix are needed (This being the main working area) print statements can be inserted at the appropriate points.

APPENDIX III

PROGRAM LISTING

BRANCH AND BOUND PROCEDURE FOR WAREHOUSE LOCATION PROBLEM

```
$IBJOB
$IBFTC MAIN
C
C
C
C      M=NUMBER OF POSSIBLE WAREHOUSE SITES
C
C      N=TOTAL NUMBER OF CUSTOMERS(DEMAND POINTS)
C
C      R(J)=DEMAND OF CUSTOMER J A NORMALLY DISTRIBUTED RANDOM VARIABLE
C
C      AMU(J)=MEAN OF THE RANDOM VARIABLE R(J)
C
C      SIGMA(J)=STANDARD DEVIATION OF THE RANDOM VARIABLE R(J)
C
C      X(I,J)=FRACTION OF THE DEMAND OF CUSTOMER I WHICH IS SATISFIED BY
C      THE WAR OUSE LOCATED AT SITE J,   I=1,----,M,   J=1,----,N.
C
C      F(I)=FIXED CHARGE RESULTING FROM ESTABLISHING A WAREHOUSE AT I
C
C      C(I,J)=COST OF SUPPLING THE ENTIRE DEMAND OF CUSTOMER J FROM A
C      WAREHOUSE LOCATED AT SITE I
C
C      Y(I)=1   IF A WAREHOUSE IS LOCATED AT SITE I   0 OTHERWISE
C
C      NI(I)=THE SET OF WAREHOUSES THAT CAN SUPPLY CUSTOMER J
C
C      P(J)=MINIMUM LEVEL OF CONFIDENCE WITH WHICH J TH.  CUSTOMER S
C      ACTUAL DEMAND MUST BE SAISFIED
C
C      D(J)=DEMAND OF CUSTOMER J WHICH ACTUALLY SATISFIED
C
C      G(I)=F(I) WHEN KK(I)IS EQUAL TO 0,0 IF KK(I) IS EQUAL TO 1
C
C      XP(J)=ABSCISSA ASSOCIATED WITH THE  LEFT TAIL OF THE NORMAL
C      PROBABILTY DENSITY FUNCTION FOR THE CONFIDENCE LEVEL P(J)
C
C
C
      COMMON NODE,HC,OPEN
      DIMENSION AC(50),AMU(100 ),C(50,100 ),CC(50,100 ),D(100),DEL(50),
     2OMEGA(50),PHI(50),PHJ(50),ZTEMP(8)
      LOGICAL NODE,OPEN
      REAL LARGE
      NODE=.TRUE.
      HC=1.0E30
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
C
C         I N P U T     D A T A
C
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
```

BRANCH AND BOUND PROCEDURE FOR WAREHOUSE LOCATION PROBLEM

```
      READ(5,100)NPROB
  100 FORMAT(I2)
      DO 11 NPROB=1,NPROB
      READ(5,101)M,N
  101 FORMAT(I3,I4)
      READ(5,102)(P(J),J=1,N)
  102 FORMAT(10F4.2)
      READ(5,103)(AMU(J),J=1,N)
  103 FORMAT(10F7.3)
      READ(5,104)(SIGMA(J),J=1,N)
  104 FORMAT(10F7.3)
      READ(5,105)(F(I),I=1,M)
  105 FORMAT(10F6.2)
      READ(5,106)((C(I,J),I=1,M),J=1,N)
  106 FORMAT(10E7.1)
CXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXXX
      CALL COST (F,C,D,P,AMU,SIGMA,XP,CC,M,N,KK)
      CALL TIME (MFL)
      NRULE=1
      IF(NRULE.EQ.1)  GO TO  5001
 5001 WRITE(6,5009)
 5009 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY LARGEST DELTA RULE*)
      WRITE(6,5017)
 5017 FORMAT(35X,56(1H-))
   22 CALL CTSIM (KK,F,OMEGA,AC,DEL,C,M,N)
      IF(NRULE.LE.4)  OPEN=.TRUE.
      IF(NRULE.GT.4)  OPEN=.FALSE.
      DO 23 I=1,M
      IF(KK(I).EQ.0)  GO TO  24
   23 CONTINUE
      GO TO 70
   24 II=0
      CALL NRULES (OMEGA,DEL,D,Y,X,G,CC,C,F,KK,PHI,NRULE,M,N,PHJ)
      DO 30 I=1,M
      IF(KK(I).NE.0)  GO TO 30
      II=1
   30 CONTINUE
      IF(II.EQ.0) GO TO 70
      NODE=.FALSE.
      GO TO 22
   70 CALL PRLIN (G,X,Y,CC,C,F,KK,M,N,Z)
      WRITE(6,3001)Z
 3001 FORMAT(/,15X,*VALUE OF OBJECTIVE FUNCTION=*,E15.8)
      ZTEMP(NRULE)=Z
      DO 8 I=1,M
 4001 FORMAT(/,9X,I2,5X,8F12.1)
    8 CONTINUE
      CALL TIME (NPL)
      NRULE=NRULE+1
      IF(NRULE.EQ.2)  WRITE(6,5010)
      IF(NRULE.EQ.3)  WRITE(6,5011)
```

```
      IF(NRULE.EQ.4)  WRITE(6,5012)
      IF(NRULE.EQ.5)  WRITE(6,5013)
      IF(NRULE.EQ.6)  WRITE(6,5014)
      IF(NRULE.EQ.7)  WRITE(6,5015)
      IF(NRULE.EQ.8)  WRITE(6,5016)
 5010 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY LARGEST OMEGA RULE*)
 5011 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY LARGEST DEMAND RULE*)
 5012 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY LARGEST Y RULE*)
 5013 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY  SMALLEST DELTA RULE*)
 5014 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY  SMALLEST OMEGA RULE*)
 5015 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY  SMALLEST DEMAND RULE*)
 5016 FORMAT(/,40X,*PROBLEM IS BEING SOLVED BY  SMALLEST Y RULE*)
      WRITE(6,5017)
      NODE=.TRUE.
      DO 75 I=1,M
      KK(I)=0
      DO 75 J=1,N
   75 C(I,J)=CC(I,J)
      IF(NRULE.GT.8)  GO TO 80
      GO TO 22
   80 CALL AMINCE (ZTEMP)
      CALL TIME (NTPC)
   11 CONTINUE
      STOP
      END
$IBFTC COST
C***************************************************************************
C
C      SUBROUTINE COST FOR CALCULATING C(I,J) MATRIX
C
C***************************************************************************
      SUBROUTINE COST (F,C,D,P,AMU,SIGMA,XP,CC,M,N,KK)
      DIMENSION C(M,N),D(N),P(N),AMU(N),SIGMA(N),XP(N),F(M),CC(M,N),
     1KK(M)
      DOUBLE PRECISION ZZ,DISTT
      CODE=1.0
      DO 10 J=1,N
      DISTT=P(J)
      CALL NORMAL(ZZ,DISTT,CODE)
      XP(J)=ZZ
      D(J)=SIGMA(J)*XP(J)+AMU(J)
   10 CONTINUE
      DO 6 I=1,M
    6 CONTINUE
      DO  11J=1,N
      DO  11  I=1,M
      C(I,J)=C(I,J)*D(J)
   11 CONTINUE
      DO 5 I=1,M
    5 KK(I)=0
      WRITE(6,2013)
```

```
 2013 FORMAT(/,56X,*STATUS OFWAREHOUSES*/,56X,19(1H-))
      WRITE(6,2015)(KK(I),I=1,M)
 2015 FORMAT(/,49X,5I6)
      DO 600 I=1,M
      DO 600 J=1,N
  600 CC(I,J)=C(I,J)
      RETURN
      END
$IBFTC NORMAL
C**********************************************************************
C       SUBROUTINE NORMAL FOR CALCULATING THE VALUES OF ABSCISSA OF NORMAL
C       DISTRIBUTION  FOR   GIVEN VALUES OF CONFIDENCE LEVEL
C**********************************************************************
      SUBROUTINE NORMAL(ZZ,DISTT,CODE)
      DOUBLE PRECISION ZZ,DISTT,Z,DIST,ALPHA
      T=0.01
      A=0.0
      Z=0.0D00
      DIST=0.5D00
C*****CODE=1.0 MEANS THAT DISTRIBUTION IS GIVEN AND Z TO BE COMPUTED
      IF(CODE.EQ.1.0) GO TO 7
C*****PART-I  GIVEN Z.DISTRIBUTION TO BE FOUND
      IF(ZZ.EQ.0.0) GO TO 100
   10 IF((ZZ-Z).GE.0.01) GO TO 1
      T=ZZ-Z
    1 ALPHA=(T/12.0)*(DF(Z)+4.0*DF(Z+0.25*T)+2.0*DF(Z+0.5*T)+4.0*DF(Z+0.
     175*T)+DF(Z+T))
      Z=Z+T
      DIST=DIST+ALPHA
      IF(ZZ.EQ.Z) GO TO 100
      GO TO 10
  100 DISTT=DIST
      GO TO 555
C*****PART-II DISTRIBUTION GIVEN , Z TO BE FOUND
    7 IF(DISTT.LT.0.5) GO TO 77
      IF(DISTT.EQ.0.5) GO TO 1000
   11 ALPHA=(T/12.0)*(DF(Z)+4.0*DF(Z+0.25*T)+2.0*DF(Z+0.5*T)+4.0*DF(Z+0.
     175*T)+DF(Z+T))
      Z=Z+T
      DIST=DIST+ALPHA
      IF(DISTT.LT.DIST) GO TO 101
      IF(DISTT.EQ.DIST) GO TO 102
      GO TO 11
C******COMPARING T WITH ACCURACY DESIRED
  101 IF(T.EQ.0.0001) GO TO 102
      Z=Z-T
      DIST=DIST-ALPHA
C*****SPECIFY ACCURACY
      T=0.0001
      GO TO 11
   77 DISTT=DISTT+0.5D00
```

```
      A=1.0
      GO TO 11
  102 ZZ=Z
      IF(A.EQ.1.0) ZZ=-ZZ
      IF(A.EQ.1.0) DISTT=DISTT-0.5D00
      GO TO 555
 1000 ZZ=0.0
  555 RETURN
      END
$IBFTC DF
      FUNCTION DF(Z)
C******NORMAL DENSITY FUNCTION
      DOUBLEPRECISION Z
      DF=DEXP(-(Z**2/2.0D00))/DSQRT(44.0D00/7.0D00)
      RETURN
      END
$IBFTC CTSIM
C
C*****************************************************************************************
C     SUBROUTINE CTISM FOR CYCLE THROGH SIMPLIFICATION CRITERIA AT A NODE
C
C*****************************************************************************************
      SUBROUTINE CTSIM (KK,F,OMEGA,AC,DEL,C,M,N)
      COMMON NODE,HO,OPEN
      LOGICAL NODE ,OPEN
      DIMENSION KK(M),F(M),OMEGA(M),AC(M),DEL(M),C(M,N)
 2013 FORMAT(/,56X,*STATUS OFWAREHOUSES*/,56X,19(1H-))
 2015 FORMAT(/,49X,5I6)
      IF (NODE) GO TO 1
      IF (OPEN) GO TO 2
      GO TO 3
C---------------------------------------------------------------------------
C     CRITERION APPLIED FOR FIX OPENING OF A WAREHOUSE
C---------------------------------------------------------------------------
    1 DO 10  I=1,M
      IF(KK(I).NE.0)  GO TO 10
      SUM=0.0
      DO  20 J=1,N
      DO  30  K=1,M
      IF(K.EQ.I)  GO TO 22
      IF(KK(I).EQ.(-1))  GO TO 22
      IF(C(K,J).GE.1.0E25)  GO TO 22
      AC(K)=C(K,J)-C(I,J)
      IF(AC(K).LT.0.0)  GO TO 41
      GO TO 30
   22 AC(K)=1.0E25
   30 CONTINUE
      MM=M-1
      AMIN=AC(1)
      DO 35  K=1,MM
      IF(AC(K+1).LT.AMIN)  GO TO 38
```

```
      GO TO 35
   38 AMIN=AC(K+1)
   35 CONTINUE
      GO TO 23
   41 AMIN=0.0
   23 SUM=SUM+AMIN
   20 CONTINUE
      DEL(I)=SUM-F(I)
      WRITE(6,2020)I,DEL(I)
 2020 FORMAT(/,50X,I3,F20.3)
   10 CONTINUE
      DO 40  I=1,M
      IF(KK(I).NE.0)  GO TO 40
      IF(DEL(I).GT.0.0)  GOTO 21
   40 CONTINUE
      RETURN
   21 DO  54  I=1,M
      IF(KK(I).NE.0)  GO TO 54
      IF(DEL(I).LE.0.0)  GO TO 54
      KK(I)=1
   54 CONTINUE
      WRITE(6,2013)
      WRITE(6,2015)(KK(I),I=1,M)
    2 DO 55 I=1,M
      IF(KK(I).EQ.0)  GO TO 60
   55 CONTINUE
      RETURN
C-------------------------------------------------------------------------
C     CRITERION APPLIED FOR REDUCING THE NO. OF NI
C-------------------------------------------------------------------------
   60 DO  70  I=1,M
      IF(KK(I).NE.1)  GO TO 70
      DO  80  J=1,N
      BMIN=C(I,J)
      DO 90  K=1,M
      IF(C(K,J).LT.BMIN)  GO TO 80
   90 CONTINUE
      DO  100  K=1,M
      IF(KK(K).EQ.1)  GO TO 100
      C(K,J)=HC
  100 CONTINUE
   80 CONTINUE
   70 CONTINUE
      DO 110 I=1,M
      DO 120 J=1,N
      IF(C(I,J).LT.1.0E25)  GO TO 110
  120 CONTINUE
      KK(I)=-1
  110 CONTINUE
      DO 130 I=1,M
      IF(KK(I).EQ.0)  GO TO 140
```

```
  130 CONTINUE
      RETURN
C-------------------------------------------------------------------
C     CRITERION APPLIED FOR FIX CLOSING  OF A WAREHOUSE
C-------------------------------------------------------------------
  140 DO 150  I=1,M
      IF(KK(I).NE.0)  GO TO 150
      SUM=0.0
      DO 160  J=1,N
      DO 170  K=1,M
      IF(KK(K).NE.1)  GO TO 152
      IF(C(K,J).GE.1.0E25)  GO TO 152
      AC(K)=C(K,J)-C(I,J)
      IF(AC(K).LT.0.0)   GO TO 162
      GO TO 170
  152 AC(K)=1.0E25
  170 CONTINUE
      MM=M-1
      CMIN=AC(1)
      DO 180 K=1,MM
      IF(AC(K+1).LT.CMIN)  GO TO 158
      GO TO 180
  158 CMIN=AC(K+1)
  180 CONTINUE
      GO TO 183
  162 CMIN=0.0
  183 SUM=SUM+CMIN
  160 CONTINUE
      OMEGA(I)=SUM-F(I)
 2023 FORMAT(/,50X,I3,F20.3)
  150 CONTINUE
      DO  138 I=1,M
      IF(KK(I).NE.0)  GO TO 138
      IF(OMEGA(I).LT.0.0)  GO TO 143
  138 CONTINUE
      RETURN
  143 DO 144  I=1,M
      IF(KK(I).NE.0)  GO TO 144
      IF(OMEGA(I).GE.0.0)  GO TO 144
      KK(I)=-1
      DO  167  J=1,N
      C(I,J)=HC
  167 CONTINUE
  144 CONTINUE
    3 DO 202  I=1,M
      IF(KK(I).NE.(-1))  GO TO 202
      DO  204 J=1,N
  204 C(I,J)=HC
  202 CONTINUE
      WRITE(6,2013)
      WRITE(6,2015)(KK(I),I=1,M)
```

```
      DO 7 I=1,M
    7 CONTINUE
      DO 203 I=1,M
      IF(KK(I).EQ.0)  GO TO 201
  203 CONTINUE
      RETURN
C-----------------------------------------------------------------------
C     FURTHER SIMPLIFICATION FOR REDUCING THE NO. OF  N(I)
C-----------------------------------------------------------------------
  201 DO 270 I=1,M
      IF(KK(I).NE.1)  GO TO 270
      DO 280 J=1,N
      DMIN=C(I,J)
      DO 290 K=1,M
      IF(C(K,J).LT.DMIN) GO TO 280
  290 CONTINUE
      DO 481 K=1,M
      IF(KK(K).EQ.1)  GO TO 481
      C(K,J)=HC
  481 CONTINUE
  280 CONTINUE
  270 CONTINUE
      GO TO 1
      END
$IBFTC PRLIN
C***********************************************************************
C
C     SUBROUTINE PRLIN FOR SOLUTION OF LINEAR PROGRAMMING
C
C***********************************************************************
      SUBROUTINE PRLIN(G,X,Y,CC,C,F,KK,M,N,Z)
      COMMON NODE,HC,OPEN
      LOGICAL NODE ,OPEN
      DIMENSION G(M),X(M,N),Y(M),CC(M,N ),F(M),KK(M),C(M,N)
      DO 15 J=1,N
      DO 16 I=1,M
      IF(C(I,J).LT.1.0E25)  GO TO 15
   16 CONTINUE
      WRITE(6,7)
    7 FORMAT(20X,*SOLUTION IS INFESIBLE*)
      Z=HC
      RETURN
   15 CONTINUE
      DO 10 I=1,M
      IF(KK(I).EQ.(-1))  GO TO 10
      IF(KK(I).EQ.1) G(I)=0.
      IF(KK(I).EQ.0) G(I)=F(I)
   10 CONTINUE
      DO  20  J=1,N
      EMIN=HC
      DO  30  K=1,M
```

```
      IF(KK(K).EQ.(-1))  GO TO 30
      CALL NCUST (NI,K,C,M,N)
      IF((CC(K,J)+G(K)/FLOAT(NI)).GE.EMIN)  GO TO 30
      KWARE=K
      EMIN=CC(K,J)+G(K)/FLOAT(NI)
   30 CONTINUE
      X(KWARE,J)=1.0
      DO 40  I=1,M
      IF(I.EQ.KWARE)  GO TO 40
      X(I,J)=0.0
   40 CONTINUE
   20 CONTINUE
      Z=0.0
      DO 50 I=1,M
      IF(KK(I).EQ.(-1))  Y(I)=0.0
      IF(KK(I).EQ.1)  Y(I)=1.0
      IF(KK(I).EQ.0)  GO TO 55
      GO TO 65
   55 ASUM=0.0
      DO  60  L=1,N
   60 ASUM=ASUM+X(I,L)
      CALL NCUST (NJ,I,C,M,N)
      Y(I)=ASUM/FLOAT(NJ)
   65 DO 70 J=1,N
   70 Z=Z+CC(I,J)*X(I,J)
      Z=Z+F(I)*Y(I)
   50 CONTINUE
      RETURN
      END
$IBFTC NCUST
C-----------------------------------------------------------------------
C      SUBROUTINE NCUST FOR CALCULATING THE NO.OF NI(I)
C----------------------------------------------------------------------------
      SUBROUTINE NCUST (NZ,KL,C,M,N)
C----------------------------------------------------------------------------
      COMMON NODE,HC,OPEN
      LOGICAL NODE,OPEN
      DIMENSIONC(M,N)
      NZ=0
      DO 10 J=1,N
      IF(C(KL,J).LT.HC)  NZ=NZ+1
   10 CONTINUE
      RETURN
      END
$IBFTC NRULES
C***********************************************************************
C
C      SUBROUTINE NRULES FOR APPLICATION OF BRANCHING RULES
C
C***********************************************************************
      SUBROUTINE NRULES (OMEGA,DEL,D,Y,X,G,CC,C,F,KK,PHI,NRULE,M,N,PHJ)
```

BRANCH AND BOUND PROCEDURE FOR WAREHOUSE LOCATION PROBLEM

```
      DIMENSION PHI(M),OMEGA(M),DEL(M),D(N),Y(M),X(M,N),G(M),CC(M,N),
     1C(M,N),F(M),KK(M),PHJ(M)
      INTEGER RULE
      DO 10RULE=NRULE,8
      IF(RULE.EQ.1.OR.RULE.EQ.5)  CALL CYC(DEL,PHI,M,N,KK,RULE)
      IF(RULE.EQ.2.OR.RULE.EQ.6)  CALL CYC (OMEGA,PHI,M,N,KK,RULE)
      IF(RULE.EQ.3.OR.RULE.EQ.7)  CALL CYC(D,PHI,M,N,KK,RULE)
      IF(RULE.EQ.4.OR.RULE.EQ.8)  GO TO 20
   15 IF(RULE.LE.4)  CALL AMAX (PHI,KK,M,N,RULE,C,PHJ)
      IF(RULE.GT.4)  CALL AMIN (PHI,KK,M,N,RULE,C,PHJ)
      RETURN
   20 CALL PRLIN (G,X,Y,CC,C,F,KK,M,N,Z)
      CALL CYC(Y,PHI,M,N,KK,RULE)
      GO TO 15
   10 CONTINUE
      END
$IBFTC CYC
      SUBROUTINE CYC (PSI,PHI,M,N,KK,RULE)
      DIMENSION PSI(M),PHI(M),KK(M)
      INTEGER RULE
      IF (RULE.EQ.3.OR.RULE.EQ.7) GO TO 15
      DO 10 I=1,M
      IF (KK(I).NE.0)  GO TO10
      PHI(I)=PSI(I)
   10 CONTINUE
      GO TO 20
   15 DO 18 I=1,M
   18 PHI(I)=PSI(I)
   20 RETURN
      END
$IBFTC AMAX
C-----------------------------------------------------------------------
C     BRANCHING RULES FOR FIXED OPENING OF A WAREHOUSE
C-----------------------------------------------------------------------
      SUBROUTINE AMAX(PHI,KK,M,N,RULE,C,PHJ)
      DIMENSION PHI(M),KK(M),C(M,N),PHJ(M)
      INTEGER RULE
      IF(RULE.NE.3)  GO TO 7
      DO 6 I=1,M
      PHJ(I)=0.0
      IF(KK(I).NE.0)  GO TO 6
      DO 5 J=1,N
      IF(C(I,J).GT.1.0E25)  GO TO 5
      PHJ(I)=PHJ(I)+PHI(J)
    5 CONTINUE
    6 CONTINUE
      DO 12 I=1,M
   12 PHI(I)=PHJ(I)
    7 RMAX=-1.0E36
      II=0
      DO 10 I=1,M
```

```
      IF (KK(I).NE.O)  GC TC 10
      IF(PHI(I).GE.RMAX) GC TC 15
      GC TC 1C
   15 RMAX=PHI(I)
      II=I
   10 CCNTINUE
      IF(II.EQ.C) WRITE(6,2C)
      KK(II)=1
      RETURN
   2C FCRMAT(1HC//*ALL VALUES LESS THAN -1.0E36*)
      END
$IBFTC AMIN.
C-----------------------------------------------------------------------
C     BRANCHING RULES FCR FIXED CLCSING OF A WAREHOUSE
C-----------------------------------------------------------------------
      SUBRCUTINE AMIN (PHI,KK,M,N,RULE,C,PHJ)
      DIMENSICN PHI(M),KK(M) ,C(M,N),PHJ(M)
      INTEGER RULE
      IF(RULE.NE.7)  GC TC 7
      DC 6 I=1,M
      PHJ(I)=0.C
      IF(KK(I).NE.0)  GC TC 6
      DC 5 J=1,N
      IF(C(I,J).GT.1.CE25)  GC TC 5
      PHJ(I)=PHJ(I)+PHI(J)
    5 CCNTINUE
    6 CCNTINUE
      DC 12 I=1,M
   12 PHI(I)=PHJ(I)
    7 RMIN=1.0E36
      HC=1.0E3C
      II=C
      DC 1C I=1,M
      IF(KK(I).NE.0)  GC TC 10
      IF(PHI(I).LE.RMIN) GC TC 15
      GC TC 10
   15 RMIN=PHI(I)
      II=I
   10 CCNTINUE
      IF(II.EQ.C) PRINT 20
      IF(II.EQ.C) WRITE(6,2C)
      KK(II)=-1
      DC 25 J=1,N
   25 C(II,J)=HC
   2C FCRMAT(1HC//*ALL VALUES GT 1.0E36*)
      RETURN
      END
 IBFTC AMINCB
C-----------------------------------------------------------------------
C     SUBRCUTINE AMINCB FCR FINDING MINIMUM CBJECTIVE FUNCTION VALUES
C-----------------------------------------------------------------------
```

```
      SUBROUTINE AMINOB(ZTEMP)
      DIMENSION ZTEMP(8),ITEMP(8)
      RMIN=1.0E36
      DO 10 L=1,8
      IF(ZTEMP(L).GT.RMIN)  GO TO 9
      RMIN=ZTEMP(L)
      ITEMP(L)=L
      GO TO 10
    9 ITEMP(L)=0
   10 CONTINUE
      WRITE(6,20)RMIN
   20 FORMAT(1H//,20X,*MINMUM VALUE OF OBJECTIVE FUNCTION IS ==*,2X,E15.
     13)
      WRITE(6,55)
      DO 50 L=1,8
      IF(ITEMP(L).EQ.0)  GO TO 50
      WRITE(6,25)ITEMP(L)
   25 FORMAT(1H//,20X,I2)
   50 CONTINUE
   55 FORMAT(/,20X,*THE FOLLOWING RULES GIVES THE MINIMUM OBJECTIVE FUNCTI
     1ON*)
      RETURN
      END
$ENTRY
```